A Markov Chain Monte Carlo for Galactic Cosmic Ray Physics

Putze et al.

[arXiv:0808.2437v1 [astro-ph]]

[A&A submitted]

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Motivation

Determination of propagation parameters

- transport in turbulent magnetic fields
- cosmic-ray sources
- γ-ray diffuse emission
- indirect dark matter detection

Analysis of secondary-to-primary ratios: new data

- PAMELA
- CREAM
- ...

Need for sound numerical tool: MCMC

Covers efficiently the parameter space
Handles multi-dimensional parameter space with minimal computing time
Used for cosmological parameter estimates: Why not for CRs?

Principle

Model depending on m parameters: $\boldsymbol{\theta} = \{\theta^{(1)}, \theta^{(2)}, \ldots, \theta^{(m)}\}$

$\Downarrow$

Bayes' theorem (Probability interpreted as a measure of degree of belief)

$$\underbrace{P(\boldsymbol{\theta}|\text{data})}_{\text{posterior probability}} \propto \underbrace{P(\text{data}|\boldsymbol{\theta})}_{\text{likelihood}} \cdot \underbrace{P(\boldsymbol{\theta})}_{\text{prior probability}}$$

$\Downarrow$

Sampling parameter space to obtain $P(\boldsymbol{\theta}|\text{data})$ with a MCMC

Exploring any target distribution $p(\boldsymbol{\theta})$ by generating a chain of n points:

$$\{\boldsymbol{\theta}_i\}_{i=1,\ldots,n} \equiv \{\boldsymbol{\theta}_1, \boldsymbol{\theta}_2, \ldots, \boldsymbol{\theta}_n\}$$

$\Downarrow$

Extracting $P(\theta^i|\text{data})$ by marginalisation

Metropolis-Hastings algorithm

Principle

Jumping from current point $\boldsymbol{\theta}_i$ in the parameter space to another $\boldsymbol{\theta}_{i+1}$ using a proposal density $q(\boldsymbol{\theta}_{i+1}|\boldsymbol{\theta}_i)$

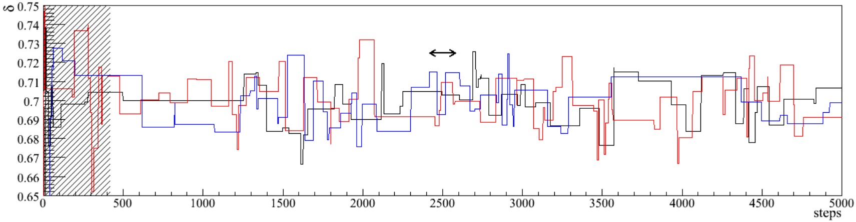
N.B.: the new point depends only on the current one:
$\mathcal{T}(\boldsymbol{\theta}_{i+1}|\boldsymbol{\theta}_1, \ldots, \boldsymbol{\theta}_i) = \mathcal{T}(\boldsymbol{\theta}_{i+1}|\boldsymbol{\theta}_i)$

Condition assuring that the chain tends asymptotically to the target PDF

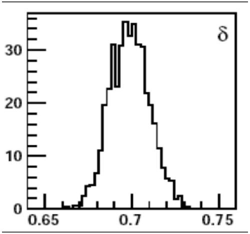
$$a(\boldsymbol{\theta}_{\rm trial}|\boldsymbol{\theta}_i) = \min\left(1, \frac{p(\boldsymbol{\theta}_{\rm trial})}{p(\boldsymbol{\theta}_i)} \frac{q(\boldsymbol{\theta}_i|\boldsymbol{\theta}_{\rm trial})}{q(\boldsymbol{\theta}_{\rm trial}|\boldsymbol{\theta}_i)}\right)$$

The trial state is accepted $(\boldsymbol{\theta}_{i+1} = \boldsymbol{\theta}_{\rm trial})$ with the probability a.
If rejected, the current state is rewritten $(\boldsymbol{\theta}_{i+1} = \boldsymbol{\theta}_i)$ in the chain.

Chain Analysis



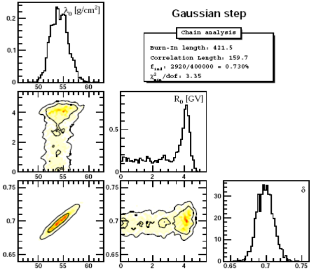
Computing burn-in and correlation length for extracting independent samples ⇒

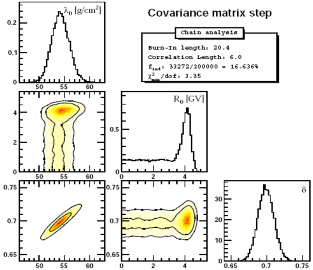


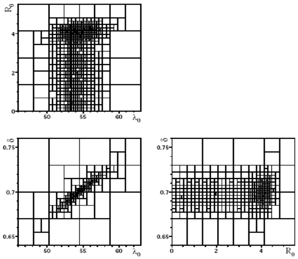
PDF of the parameter

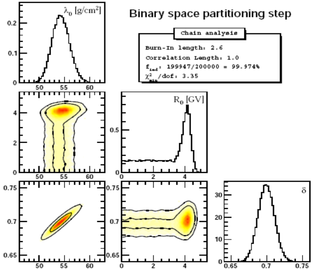
⇒ mean value and confidence intervals

Trial functions



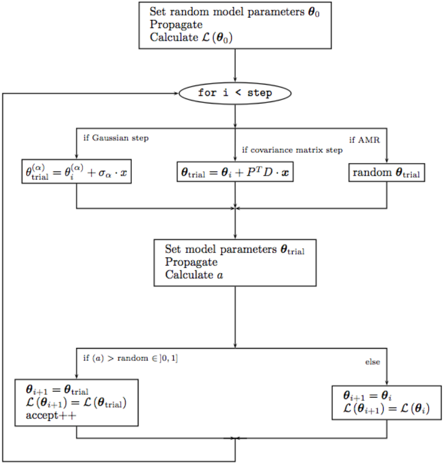






Implementation

Propagation model independent



MCMC related input

Number of parameters
Likelihood function
Trial function
Number and Length of chains

Propagation model related input

Parameters
Propagation equation & solution

User choice

Propagation model:
Leaky-Box Model
Free parameters of the model:
$\{\lambda_0, R_0, \delta_0, \delta, \mathcal{V}_a, \alpha, \eta, q_C, q_N, q_O\}$
Data used to constrain model:
B/C, CNO, antiprotons

Fitting B/C using HEAO-3 data [Engelmann et al., 1990, A&A, 233, 96]

Adjusting the grammage $\lambda_{\rm esc}(R)$:

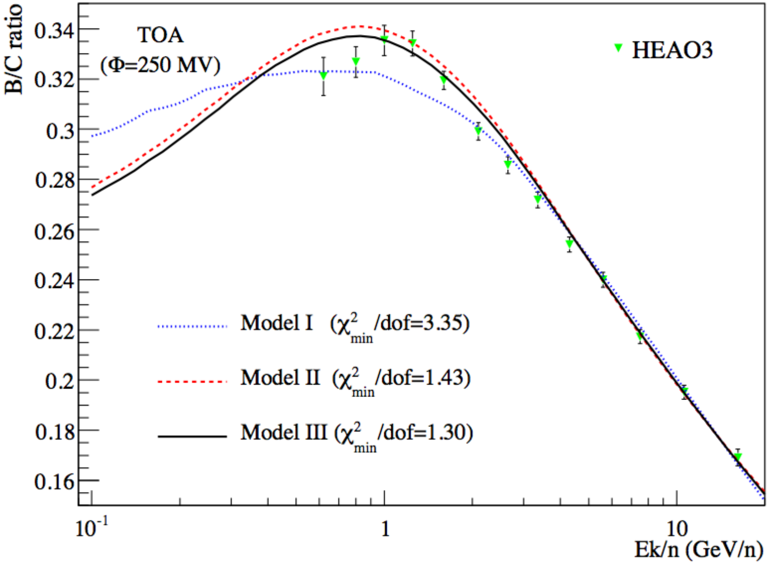
$$\lambda_{\rm esc}(R) = \begin{cases} \lambda_0 \beta {R_0}^{-(\delta-\delta_0)} R^{-\delta_0} & \text{when } R < R_0, \\ \lambda_0 \beta R^{-\delta} & \text{otherwise;} \end{cases}$$

We use a LBM with minimal reacceleration by interstellar turbulence [E. S. Seo & V. S. Ptuskin, 1994, ApJ, 431, 705]

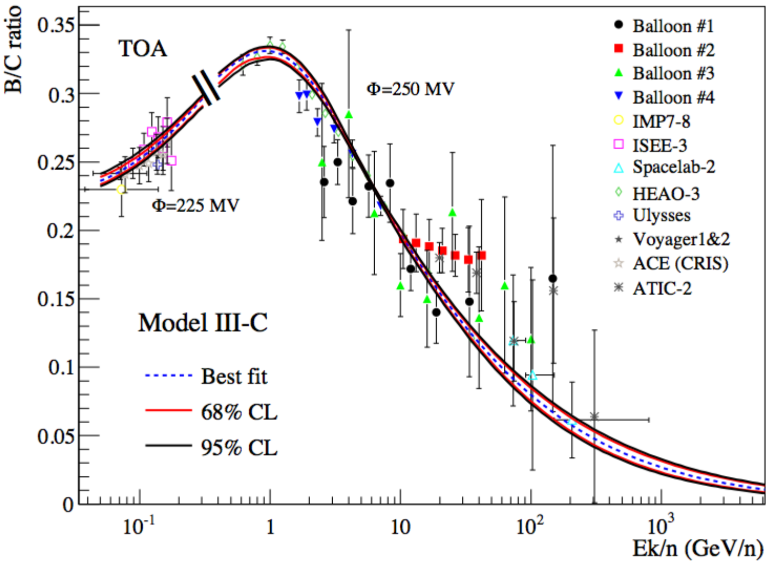
Model	λ_0 g cm^{-2}	R_0 GV	δ	$\mathcal{V}_a$ km s^{-1}kpc^{-1}	$\chi^2_{\rm min}$/dof
I	54^{+2}_{-2}	$4.2^{+0.3}_{-0.9}$	$0.70^{+0.01}_{-0.01}$	-	3.35
II	26^{+2}_{-2}	-	$0.52^{+0.02}_{-0.02}$	88^{+6}_{-11}	1.43
III	30^{+5}_{-4}	$2.8^{+0.6}_{-0.8}$	$0.58^{+0.01}_{-0.06}$	75^{+10}_{-13}	1.30

Results of Model I compatible with results of [Webber et al., 1998, ApJ, 508, 940]
Results of Model III for reacceleration in agreement with [Maurin et al., 2002, A&A, 294, 1039], in which a 2D diffusion model with reacceleration and convection was used

Best fit HEAO-3



HEAO-3+low energy: Confidence region



Fitting simultaneously B/C and O

Adjusting simultaneously the grammage $\lambda_{\rm esc}$ and the primary source spectrum $Q_j(E)$ for nuclear species j:

$$Q(E) = q_j \beta^{\eta_j} R^{-\alpha_j} \quad \text{with} \quad \alpha_j = \alpha \quad \text{and} \quad \eta_j = \eta = -1$$

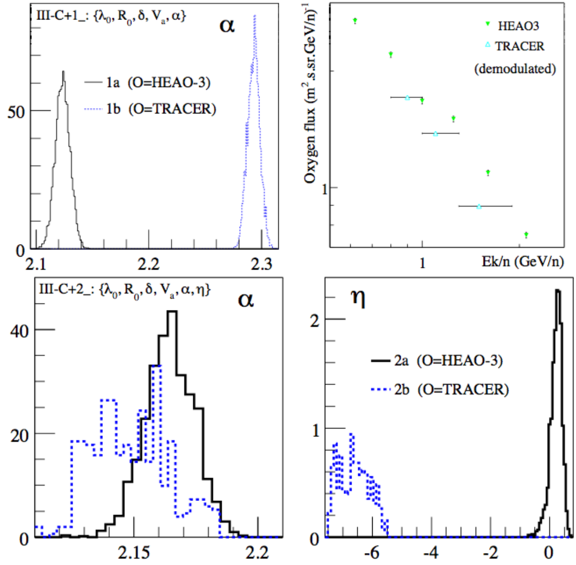
Dataset a/b: HEAO-3/TRACER [Ave et al., 2008, ApJ, 677, 262] data used for fitting O fluxes

Model	λ_0 g cm^{-2}	R_0 GV	δ	$\mathcal{V}_a$ km s^{-1}kpc^{-1}	α	η
III+1a	37^{+2}_{-2}	$4.4^{+0.1}_{-0.2}$	$0.61^{+0.01}_{-0.01}$	64^{+4}_{-4}	$2.124^{+0.005}_{-0.007}$	-
III+1b	$20.9^{+0.2}_{-0.8}$	$0.3^{+0.6}_{-0.1}$	$0.47^{+0.01}_{-0.01}$	103^{+2}_{-3}	$2.294^{+0.004}_{-0.006}$	-
III+2a	29^{+2}_{-2}	$2.7^{+0.3}_{-0.4}$	$0.55^{+0.01}_{-0.02}$	84^{+4}_{-7}	$2.16^{+0.01}_{-0.01}$	$0.3^{+0.1}_{-0.2}$
III+2b	32^{+4}_{-1}	$4.3^{+0.3}_{-0.1}$	$0.56^{+0.03}_{-0.01}$	62^{+2}_{-2}	$2.14^{+0.03}_{-0.01}$	$-6.7^{+0.9}_{-0.1}$

η absorbs uncertainties coming from either the modulation or the source spectrum low energy shape

"Unbiaised" determination of alpha $\sim$ 2.15 in the LBM

α and η PDFs



Fitting simultaneously B/C and CNO

Adjusting simultaneously the grammage $\lambda_{\rm esc}$ and the primary source spectrum $Q_j(E)$ for nuclear species j:

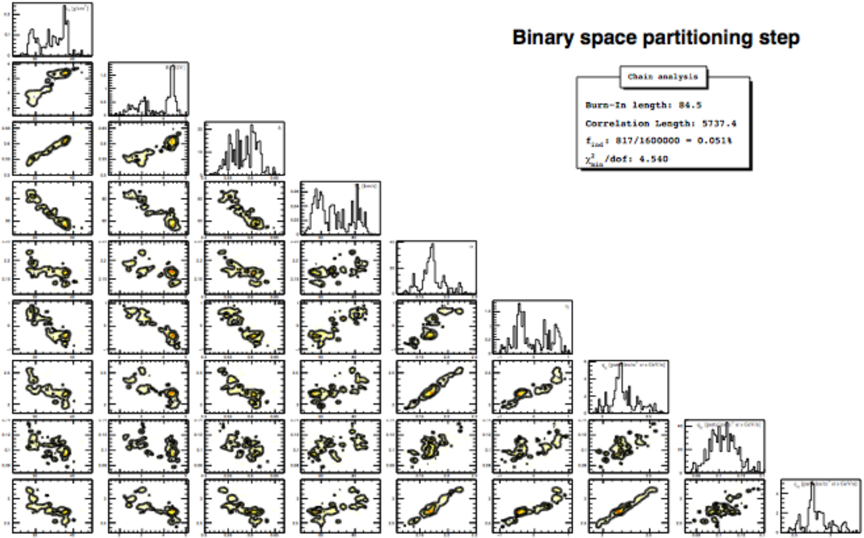
$$Q(E) = q_j \beta^{\eta_j} R^{-\alpha_j} \quad \text{with} \quad \alpha_j = \alpha \quad \text{and} \quad \eta_j = \eta = -1$$

Dataset: HEAO-3 data [Engelmann et al., 1990, A&A, 233, 96] used for fitting CNO fluxes

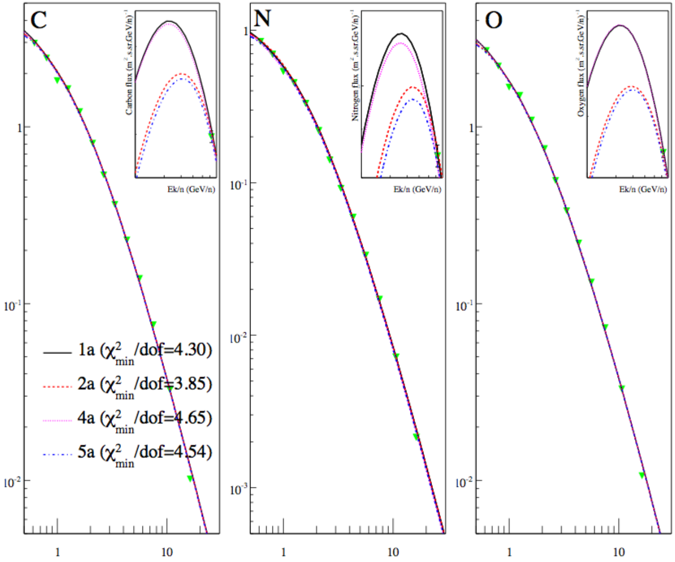
Model	α	η	$10^{20} \times (q_C \vert q_N \vert q_O)$ $(\text{m}^3 \text{ s GeV/n})^{-1}$
III+4	$2.13^{+0.01}_{-0.01}$	-	$1.93^{+0.04}_{-0.004} \vert 0.089^{+0.007}_{-0.005} \vert 2.42^{+0.04}_{-0.05}$
III+5	$2.17^{+0.02}_{-0.02}$	$-0.4^{+1.2}_{-0.1}$	$2.2^{+0.2}_{-0.1} \vert 0.107^{+0.01}_{-0.006} \vert 2.7^{+0.3}_{-0.1}$

Model III+5 provides values for the relative abundances C:N:O close to those derived from HEAO-3 [Engelmann et al., 1990, A&A, 233, 96].
Source spectrum parameters are correlated to propagation parameters.

Correlations between propagation and source parameters



CNO fluxes



C
N
O
Carbon flux (m^-2.s.sr.GeV/n)^-1
Nitrogen flux (m^-2.s.sr.GeV/n)^-1
Oxygen flux (m^-2.s.sr.GeV/n)^-1
Ek/n (GeV/n)
1a (χ^2_{min}/dof=4.30)
2a (χ^2_{min}/dof=3.85)
4a (χ^2_{min}/dof=4.65)
5a (χ^2_{min}/dof=4.54)
1
10
10^{-1}
10^{-2}
10^{-3}

Conclusion

MCMC algorithm and implementation:

- Successful implementation of an MCMC to extract the posterior PDF of the propagation parameters in a LBM
- Sequential use of three different trial functions (zoning to fine description)
- Running numerous chains in parallel

Results:

- Comparing the impact of choosing different data sets
- Model with rigidity cutoff and reacceleration preferred
- Kolmogorov spectral index of 1/3 excluded
- Model with 2 diffusion slopes is not favoured by the data
- Correlation between the propagation and source parameters, biaising their estimation

Perspectives

- At present $\bar{p}$ data do not cover a large enough energy range to constrain the propagtion parameters through a direct fit
$\Longrightarrow$ requires better statistics on the $\bar{p}$ flux especially at higher energy (new PAMELA data up to 100 GeV)
- add other primary species to constrain further α and/or check $\alpha_i \neq \alpha_j$ for different species and/or diagnose some problems in the data if we believe the slope should be universal
- fit well measured H and He species to characterise the low energy source spectra η_i
- use more realistic diffusion models on larger data sets including more nuclear species
- Taking advantage of new data