Influence of Cherenkov light on the width of optical images of extensive air showers

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Abstract— **We show that Cherenkov light has an important role in determining the width of shower light images, as measured in the EAS experiments using fluorescence technique. Its role is dominant below shower maximum. This conclusion is based on numerical calculations using universality of electron distributions (energy, angle and lateral distance) in different showers.**

I. Introduction

Extensive Air Showers (EAS) produced by cosmic ray particles cause excitation of the atoms of the atmosphere and a subsequent isotropic emission of light by nitrogen molecules, mostly in 300-400 nm wavelength range, called fluorescence light (FL). This light can be detected by earth-based telescopes. The advantage of the fluorescence technique, in comparison with ground particle detectors, is the possibility to register light from various heights of the developing shower and to calculate the number of particles vs. shower depth $N(X)$ (longitudnal profile), or, in fact, the energy deposit $\frac{dE}{dX}$ as a function of shower depth X along the whole shower profile. This can be done because the FL light emitted by a shower path element is proportional to the energy loss of shower particles along this element [1], [2].

This technique was used with success by the Fly's Eye experiment [3], by its successor HiRes [4], [5] and recently by the Pierre Auger Observatory [6].

The problem is that some fraction of shower particles produce also Cherenkov light (ChL). Although its direction is almost the same as that of the emitting particle's velocity, it can be scattered sideways in the atmosphere and registered by the FL detector. Due to the lateral spread of particles (mostly electrons and positrons) in the shower, the ChL has also some lateral distribution. This may influence the width of the shower image on detector's camera.

The goal of this work is to determine the lateral distribution of Cherenkov light at a given level of a shower development. This distribution depends on the angular and lateral distributions of this light produced along the shower down to the level observed. However, the contribution of a particular depth element of the shower depends on the atmospheric transmission from this element down to the level of interest. This transmission has to be taken into account.

Thus, we calculate here the angular and lateral distributions of Cherenkov photons produced at different levels in the atmosphere per average electron (see later). We will show that these distributions depend only on the shower age at the considered level and on its height in the atmosphere.

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II. Angular distribution

The angular distribution of Cherenkov (Ch) electrons, $G_\theta(\theta; s, h)$, at a given level h in the atmosphere, where s is the shower age, equals:

$$G_\theta(\theta; s, h) = = \frac{2\pi \sin\theta}{F(s,h)} \int_{E_{th}(h)}^{\infty} g_\theta(\theta; E) \cdot f(E; s) \cdot Y_{Ch}\left(\frac{E}{E_{th}}\right) \cdot dE \quad (1)$$

where $g_\theta(\theta; E)$ is the angular distribution of electrons with energy E [7], $f(E; s)$ is the electron energy spectrum (we used the one parametrised by Nerling et al. [8]), $Y_{Ch}(E)$ is the ratio the number of Ch photons produced by an electron with energy E along unit path (in g cm^{-2}) to their maximum number ($E \gg E_{th}$)

$$Y_{Ch}(E) = 1 - \left(\frac{E_{th}}{E}\right)^2 \quad (2)$$

and

$$F(s,h) = \int_{E_{th}(h)}^{\infty} Y_{Ch}(E)\, f(E,s)\, dE \quad (3)$$

– the effective fraction of electrons emitting Ch light [9].

The normalisation of $G_\theta(\theta; s, h)$ is such that:

$$\int_0^{\frac{\pi}{2}} G_\theta \, d\theta = 1 \quad (4)$$

The fact that photons are produced within the Cherenkov cone and not along electron's track has also been taken into account, although it affects G_θ very little.

We have parametrised $G_\theta(\theta; s, h)$ as follows [7]:

$$G_\theta(\theta; s, h) = \frac{1}{N}\frac{dN}{d\theta} = \begin{cases} a_1 \cdot e^{-(c_1\cdot\theta + c_2\cdot\theta^2)} & \text{for } \theta < \theta_0 \\ a_2 \cdot \theta^{-\alpha} & \text{for } \theta > \theta_0 \end{cases} \quad (5)$$

where the parameter's dependence on age s and height h (in km, above sea level):

$$a_i(s,h) = p_0 \cdot e^{p_1\cdot s + p_2\cdot h} + p_3 \quad (6)$$

for parameters θ_0, a_1, c_1 and α,
and:

TABLE I

VALUES OF THE FIT'S PARAMETRES FOR ANGULAR DISTRIBUTION OF CHERENKOV ELECTRONS G_θ.

p_i	θ_0	a_1	c_1	c_2	α
p_0	6.058e+00	2.905e+00	7.320e+00	-2.754e+00	2.620e+00
p_1	-1.103e-03	-3.851e-02	-3.778e-01	-4.242e-01	6.837e-02
p_2	-2.886e-03	1.072e-01	7.202e-02	1.084e-01	-2.247e-02
p_3	-5.447e+00	1.066e+01	7.143e+00	3.344e+00	1.110e+00
p_4				-4.294e+00	

TABLE II

VALUES OF THE FIT'S PARAMETERS FOR LATERAL DISTRIBUTION OF CHERENKOV ELECTRONS G_x.

p_i	α	k	β
p_0	-6.087e-04	-9.864e-01	1.206e-01
p_1	5.619e-02	1.599e-01	9.075e-03
p_2	3.428e-03	1.947e+00	-6.902e+00
p_3	-2.618e-03	7.696e-05	5.509e-01
p_4	-6.003e-02	-4.462e-03	2.757e+00
p_5	8.200e+00	6.259e-02	
p_6	1.235e+00	5.858e-01	

$$a_i(s,h) = p_0 \cdot e^{p_1 \cdot s^2 + p_2 \cdot h} + (p_3 \cdot s^2 + p_4 \cdot s) \tag{7}$$

for parameter c_2.

a_2 can be obtained from the condition of continuity of the functions at θ_0.

The values of the parameters are presented in TABLE I. Examples of the distributions with fits are presented on Fig. 1 where two upper graphs are for shower age s=1, lower graphs for s=1.2; left graphs are for height h=5 km and right – h=8 km; points are result of simulations, fits are presented by solid line.

III. LATERAL DISTRIBUTION

The lateral distribution of Cherenkov electrons, $G_r(r; s, h)$, at distance r, was calculated as:

$$\frac{1}{N_e}\frac{dN_e}{d\log r} = G_r(r;s,h) = \\ = \frac{2\pi r^2 \ln 10}{F(s,h)} \int_{E_{th}(h)}^{\infty} g_r(r;E,s,h) \cdot f(E;s) \cdot Y_{Ch}(E) \cdot \, dE \tag{8}$$

where $g_r(r; E, s)$ is the lateral distribution of electrons with energy E. It has been shown in [10] that the lateral distribution of electrons with a given energy depends on the shower age only, if the distances are expressed in the Molière units r_M. Even better unit is $r_E = \frac{\beta \, r_M}{E}$, as using it the distributions for different E become more similar to each other.

This function satisfies the normalisation condition:

$$\int_0^\infty G_r \, d\log r = 1 \tag{9}$$

It is convenient to express this distribution in units of the average root mean square of the distance $r_\perp$ for the Ch electrons rather than r, so we introduce the variable $x = \frac{r}{r_\perp(s,h)}$ (see formula 18).

We shall parametrise the new distribution $G_x(x; s, h)$ as follows:

$$G_x(x;s,h) = C \cdot x^\alpha \cdot (1+kx)^{-\beta} \tag{10}$$

where C guarantees the correct normalisation, and equals:

$$C = \ln(10) \cdot k^\alpha \frac{\Gamma(\beta)}{\Gamma(\alpha)\Gamma(\beta-\alpha)} \tag{11}$$

The dependence on s and h (in km, above sea level) is as follows:

$$\alpha(s,h) = (p_0 \cdot s^3 + p_1 \cdot s + p_2 \cdot h + p_3 \cdot s \cdot h + p_4) \cdot (h + p_5) + p_6 \tag{12}$$

$$k(s,h) = e^{p_0 \cdot s^2 + p_1 \cdot h + p_2 \cdot s} \cdot (p_3 \cdot h^2 + p_4 \cdot s \cdot h + p_5 \cdot s) + p_6 \tag{13}$$

$$\beta(s,h) = e^{p_0 \cdot s^2 \cdot h + p_1 \cdot h^2 + p_2} + p_3 \cdot s + p_4 \tag{14}$$

The values of the parameters are presented in TABLE II. Examples of the distributions with fits are shown at Fig. 2 where two upper graphs are for shower age s=1, lower graphs for s=1.2; left graphs are for height h=5 km and right – h=8 km; points are result of simulations, fits are presented by solid line; error bars represent 5% of the bin's value.

IV. AVERAGE TANGENT

The square root of average square tangent of the angle θ between the direction of Ch electron and the shower core, was calculated as:

$$\tan_{rms}\theta(s,h) \equiv \sqrt{\langle \tan^2\theta(s,h) \rangle} = \\ = \sqrt{\int_0^{\frac{\pi}{2}} \tan^2\theta \cdot G_\theta(\theta;s,h) \cdot \, d\theta} \tag{15}$$

and can be approximated by the following function:

$$\tan_{rms}\theta = p_0 \cdot e^{p_1 \cdot s + p_2 \cdot h^2 + p_3 \cdot h} \tag{16}$$

with: $p_0 = 0.1141$, $p_1 = 0.2501$, $p_2 = -5.291 \cdot 10^{-4}$ and $p_3 = -2.029 \cdot 10^{-2}$.

We also define:

$$r_\theta \equiv y \, \tan_{rms}\theta \tag{17}$$

where y is the distance between photon production level to the observation level (Fig. 3). The usefulness of $\tan_{rms}\theta$ and r_θ, as well as $r_\perp(s,h)$ (see § V) will become clear in § VI.

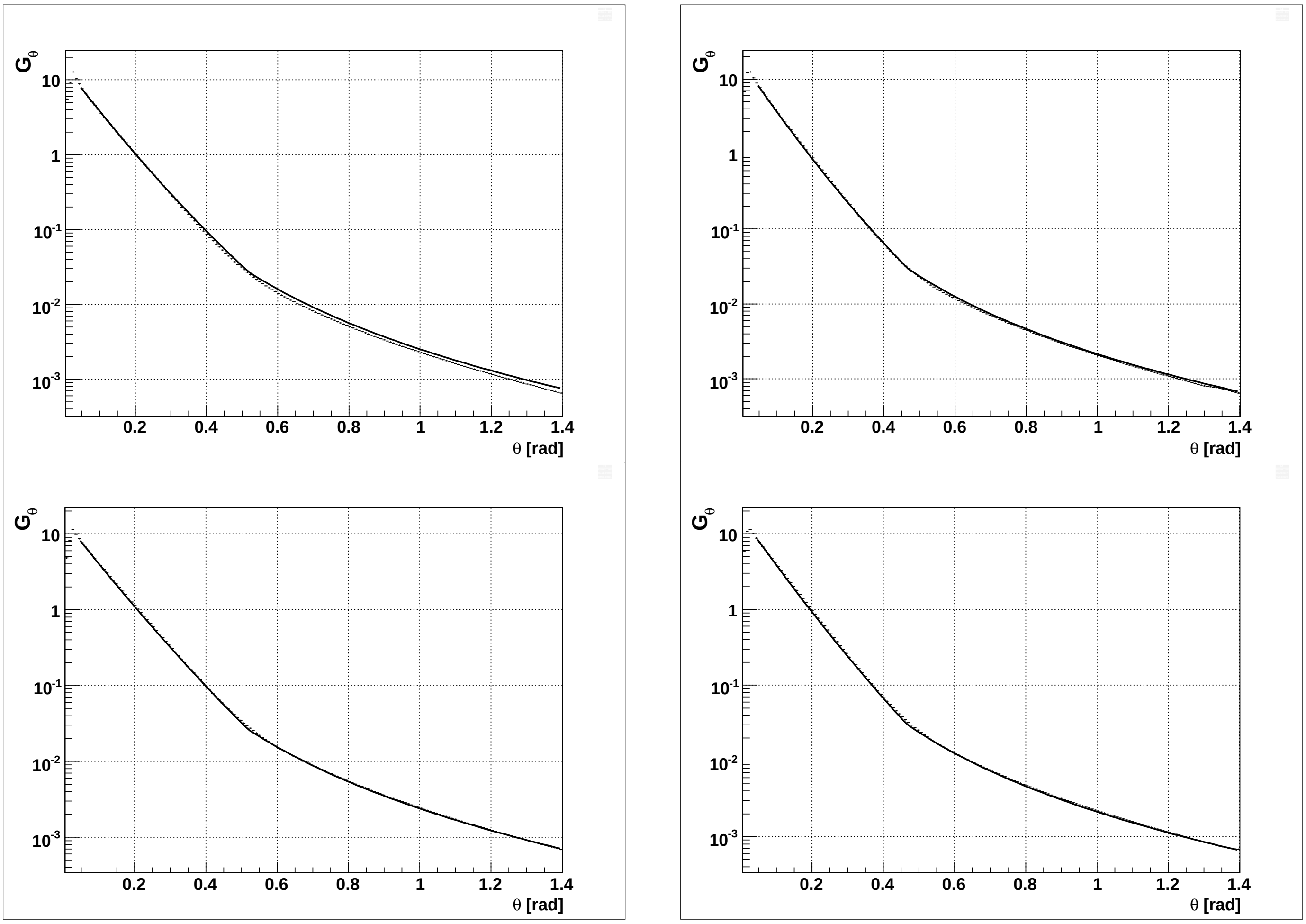

Fig. 1. Angluar distributions of Ch electrons $G_\theta(\theta; s, h)$. Two upper graphs – s=1, lower graphs – s=1.2; left graphs – h=5 km, right graphs – h=8 km. Points – simulation results, line – fit.

V. Average distance from the shower core

The square root of average square distance of the Ch electrons from the shower core equals:

$$r_\perp(s,h) \equiv \sqrt{\langle r^2(s,h)\rangle} = \\ = \sqrt{\frac{\int_{E_{th}(h)}^{\infty} \langle r^2(E;s,h)\rangle \cdot f(E;s) \cdot Y_{Ch}\left(\frac{E}{E_{th}}\right) \cdot dE}{\int_{E_{th}(h)}^{\infty} f(E;s) \cdot Y_{Ch}\left(\frac{E}{E_{th}}\right) \cdot dE}} \tag{18}$$

where $\left\langle r^2(E;s,h)\right\rangle = \left\langle \chi^2(E;s)\right\rangle \cdot r_E^2(h)$ and $\chi^2(E;s)$ were calculated analytically from the fits to $g_x(x;E,s)$ adopted in the Nishimura-Kamata form (as in 10).

We approximated $r_\perp(s,h)$ as below:

$$r_\perp(s,h) = p_0 \cdot e^{p_1 \cdot s + p_2 \cdot h} \tag{19}$$

with: $p_0 = 10.24$, $p_1 = 1.123$ and $p_2 = 0.1109$.

VI. Procedure to calculate the lateral distribution of Cherenkov photons (LDCh)

The parametrisations presented in the previous sections can be used to calculate the lateral distribution of Cherenkov photons at a given level.

LDCh at a given level of the shower development depends on the angular and lateral distributions of Ch electrons above this level. Here, we shall assume that the two distributions are independent (i.e. the angular distribution does not depend on the core distance) which is not quite true. However, we hope that for our purposes it is adequate enough.

The next step is to check which of two distributions (taken separately) at some level X would have produced a broader LDCh at X_{obs} in terms of the corresponding root mean square distances: r_θ – corresponds to LDCh resulting from the angular distribution of Ch electrons only, $r_\perp$ – from the lateral distribution only. Then, we assume that the resulting contribution from level X has LDCh following from the dominant effect (e.g. the angular distribution of electrons at X only), the second effect (here, the lateral distribution of Ch electrons at X) being taken into account by broadening the

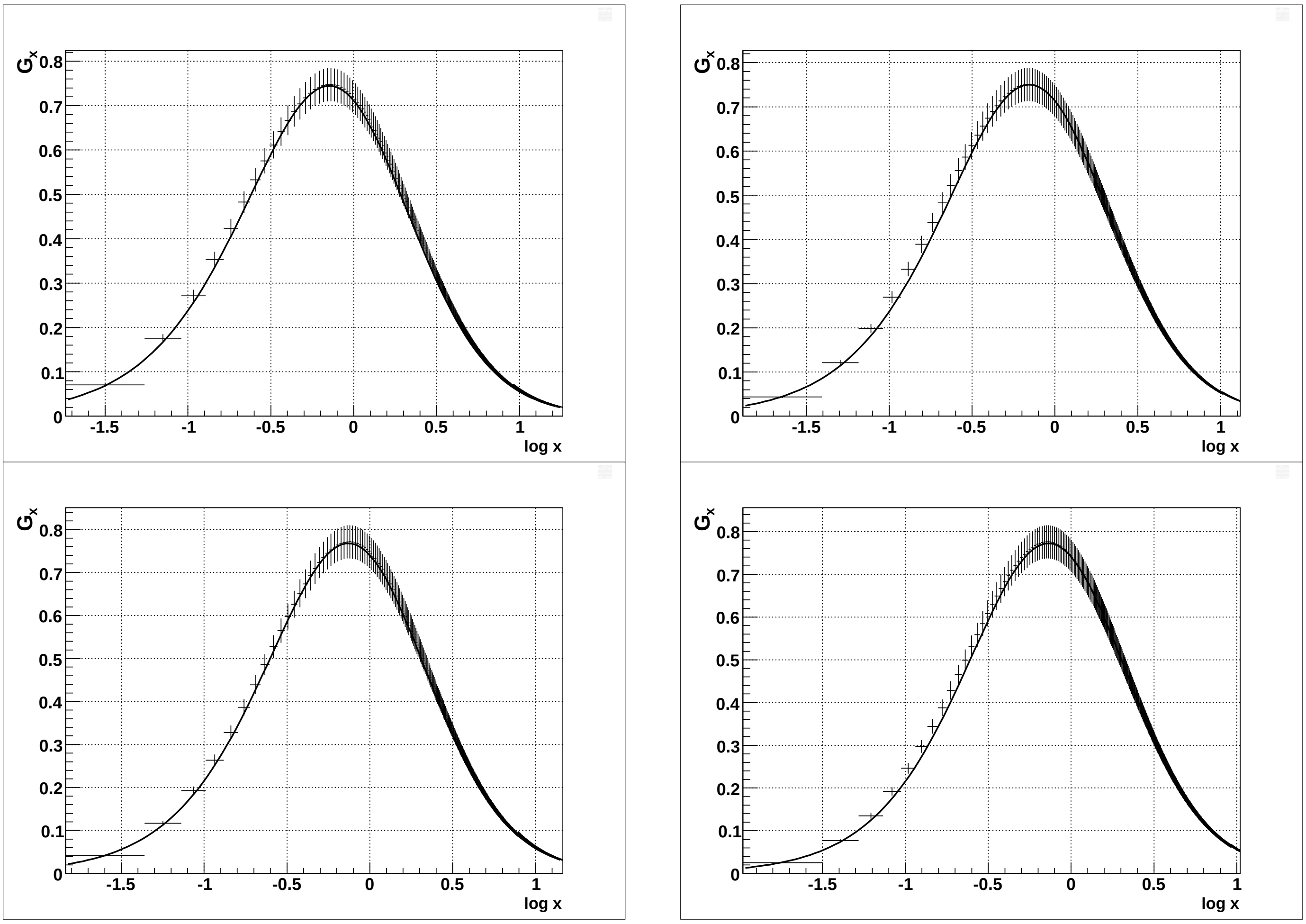

Fig. 2. Lateral distributions of Ch electrons $G_x(x; s, h)$. Two upper graphs – s=1, lower graps – s=1.2; left graphs – h=5 km, right graphs – h=8 km. Points – simulation results, line – fit. Error bars represent 5% of the value in the bin.

distribution in such a way as to get

$$\overline{r^2} = r_\theta^2 + r_\perp^2 \tag{20}$$

Thus, we have two cases:

1) $r_\theta > r_\perp$ – this concerns the upper part of the shower; the main effect is caused by the angular distribution of electrons, the lateral distribution is taken into account as a correction.
The number of Ch photons produced within angles $(\theta; \theta + \Delta\theta)$ equals:

$$d(\Delta C) = N_e\, k\, F(s,h)\, G_\theta(\theta; s,h)\, \Delta\theta \cdot dX \tag{21}$$

where k is maximum number of Ch photons produced by an electron per unit of path (172 $\frac{\text{photons}}{\text{g} \ \ \text{cm}^{-2}}$).
The radius

$$r = y\, \tan\theta \tag{22}$$

and the bin width

$$\Delta r = y\left[\tan(\theta + \Delta\theta) - \tan(\theta)\right] \tag{23}$$

are transformed to

$$r \rightarrow r\sqrt{1 + \left(\frac{r_\perp}{r_\theta}\right)^2} \equiv r' \tag{24}$$

and

$$\Delta r' \equiv \Delta r\sqrt{1 + \left(\frac{r_\perp}{r_\theta}\right)^2} \tag{25}$$

Next, the transmission factor $T(l)$ must be applied, where: $l = y\sqrt{1 + \left(\frac{r}{y}\right)^2}$.
Finally, the number of photons in the radial ring $(r', r' + \Delta r')$ equals:

$$d\Delta C(r') = T(l) \cdot d\Delta C\left(\theta[r(r')]\right) \tag{26}$$

2) $r_\theta < r_\perp$ – lower part of the shower (usually just above the observation level); now LDCh is determined by G_r, with G_θ treated as a correction by making the transformation

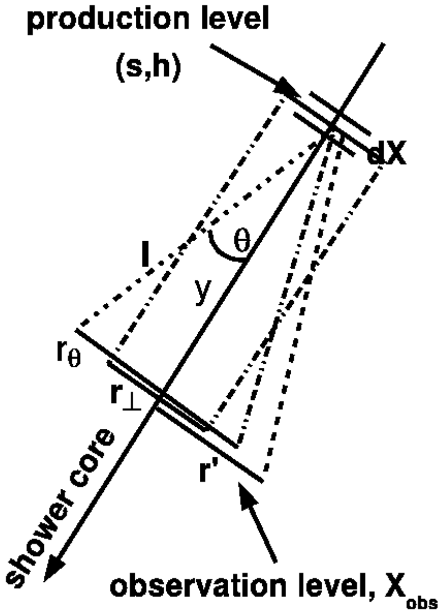

Fig. 3. Illustration of the contribution of the shower element dX, at X (production level) to LDCh at the observation level, X_{obs}. Here the angular distribution of Ch electrons results in a broader LDCh at X_{obs} than the lateral distribution of electrons ($r_\theta > r_\perp$). Thus, the final LDCh is broadened according to 24.

$$r \rightarrow r\sqrt{1 + \left(\frac{r_\theta}{r_\perp}\right)^2} \equiv r' \qquad (27)$$

and:

$$d\Delta C(r') = T(l) \cdot N_e\, k\, F(s,h)\, G_r(r[r'])\, \Delta r' \cdot dX \qquad (28)$$

Finally, to obtain LDCh at the observation level X_{obs}, we must sum up $d\Delta C(r')$ over all depth elements dX above this level:

$$\Delta LDCh(r') = \int_{X > X_{obs}} d\Delta C(r') \qquad (29)$$

VII. Results of calculations of LDCh

To calculate LDCh at a particular level in a shower it is necessary to know the attenuation and scattering properties of the atmosphere. As these depend on the season and may change from one night to another, or even within one hour or so, a constant atmosphere monitoring is needed. However, to show what will be a typical width of a shower image in ChL and compare it with that in fluorescence light it is enough to adopt typical atmosphere properties.

Here we have assumed an atmosphere describing average conditions at the Pierre Auger Observatory in Argentina (∼1450 m.a.s.l., dry climat), with the mean free path for Rayleigh scattering $\lambda_R = 18.4$ km and for aerosol (Mie) scattering $\lambda_M = 14$ km at Auger level. The exponential scale heights are correspondingly $H_R = 7.5$ km and $H_M = 1.2$ km.

In the experiments using fluorescence technique the telescopes look at showers from the side (at rather large angles to shower directions). Thus, to compare both widths (in ChL and FL) of a shower image we have to calculate the ChL scattered sideways from the shower direction. As the FL is emitted isotropically by any shower element and the Rayleigh scattering is roughly isotropic ($\sim 1 + \cos^2\theta$) we compare here numbers of Ch photons Rayleigh scattered in all directions, along 1 m of shower track, at lateral distances larger than r/r_M with numbers of FL photons emitted outside this radius along the same path.

The results of calculations are presented in Fig. 4 for three levels of shower development: $s = 0.9$, 1.1 and 1.3 for which a comparison of the lateral spread of the scattered ChL (in all directions) with that of FL is shown. The curves are for typical proton and iron showers, inclined at three zenith angles: $0°$, $45°$ and $60°$. The curve representing FL does not depend on zenith angle, neither on primary particle, as the lateral distribution of shower electrons depends on shower age only, if represented as a function of r/r_M. To calculate FL curves we have used the parametrisation of Góra et al, [11], where shower's image widths are calculated for FL only. Our results show that this is proper only for widths above (in height) shower maximum ($s < 1$). Fig. 4 shows that at $s = 0.9$ the contribution of ChL to FL is small. However, when s increases the ChL contribution increases also. For $s = 1.1$ it starts to dominate FL above one Molière radius, or so. Even deeper in the atmosphere ($s = 1.3$) it is the ChL which practically determines the lateral spread of the image. This corresponds to the Auger level for a 10^{19} eV proton shower with $z = 45°$. Thus, it can not be neglected when reconstructing the energy losses of shower particles at these levels, which are proportional to the fluorescence intensity only. The Ch photons must be taken into account not only when substracted from the total signal measured by a telescope, but at an earlier stage, when the angular width on the camera for signal selection is chosen.

VIII. Conclusion

We have shown that lateral distribution of scattered Cherenkov light may play important role in determining the width of the optical image of the shower in the fluorescence experiments, especially for lower parts of the shower, and should be included in the profile reconstruction. We have shown, that the angular and lateral distributions of Cherenkov electrons at observation level depend on shower age and height in the atmosphere of this level only. Finally, we have presented a procedure of how to calculate the amount of ChL in the light image of a shower.

Acknowledgements

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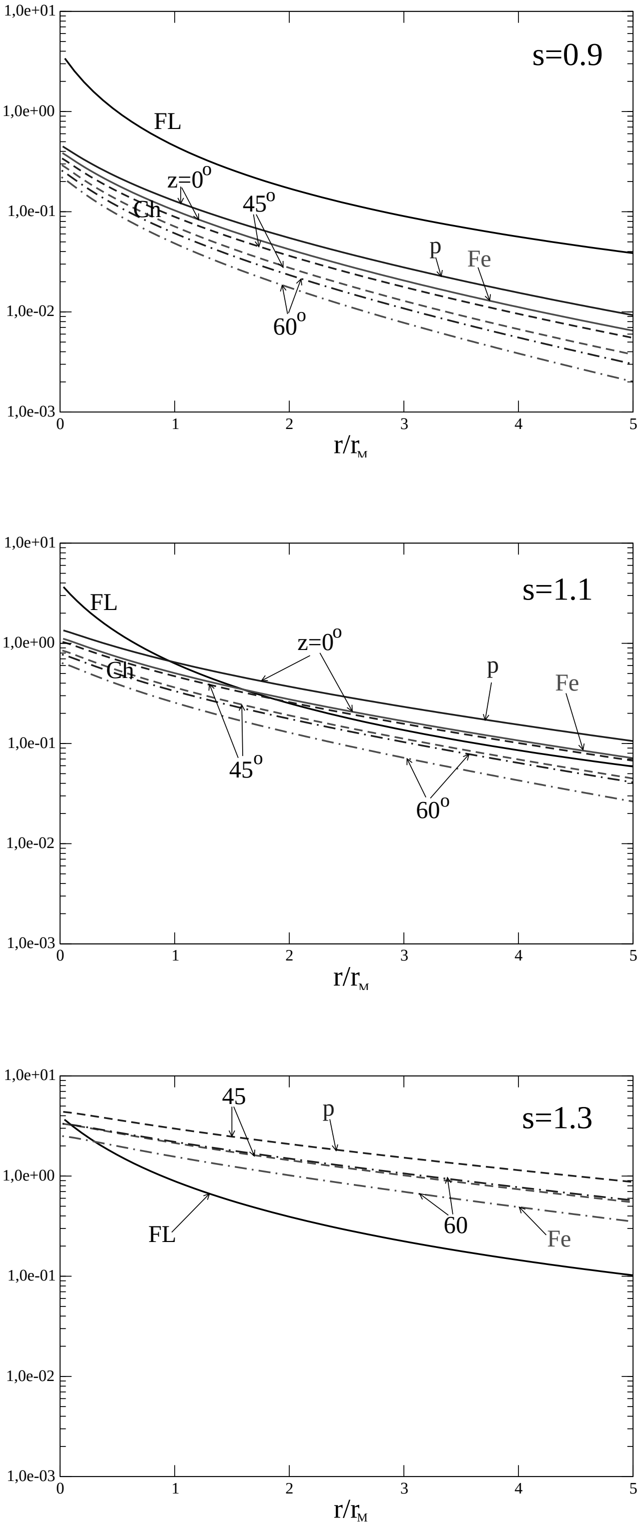

Fig. 4. Number of Ch photons scattered along a 1 m path at lateral distances larger than r, divided by total number of electrons, N_e, compared with the fluorescence light produced there (line marked FL) for three values of age: s=0.9, 1.1, 1.3 and for different zenith angles z. Curves for proton showers are higher than those for Fe; curves for larger zenith angles are lower. The importance of ChL on the lateral width of the shower light is evident below the shower maximum.

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