Coupling functions for primary cosmic rays and ground-level muons at various zenith angles

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***Abstract*—Features of coupling functions calculations for muon hodoscopes are considered. Coupling functions are calculated on the basis of simulation of cosmic ray penetration through the Earth atmosphere using CORSIKA code. Results of calculations of main coupling functions for three types of primary cosmic ray (CR) spectra (galactic CR, cosmic rays in solar proton events and variation of galactic CR during Forbush effects) are presented. Average and median energies of primary cosmic ray flux, which gives the contribution to muon detector counting rate, are calculated for different zenith angles. It is shown that even a single muon hodoscope allows to examine a wide range of primary cosmic ray energies.**

1. Introduction

ONE of the methods of the study of solar, heliospheric and geomagnetic processes is the analysis of cosmic ray variations detected by various ground-based detectors. For this purpose neutron monitors (NM), muon telescopes and muon detectors included in the arrays for detection of extensive air showers (EAS) are usually used. New possibilities for ground-based monitoring of CR variations are opened with the use of muon hodoscopes which make possible to measure the spatial-angular variations of muons flux simultaneously from all directions of the upper hemisphere [1]. However, to analyze variations in the primary CR flux by means of muon hodoscopes, coupling functions [2] which allow link variations in counting rate of ground-based setups and the changes of primary CR intensity on the border of the atmosphere are necessary. Calculations of the coupling functions were performed earlier mainly for NM (see review [3]) and for muon detectors included in EAS arrays [4-5].

In this work, the features of the method of calculation of main coupling functions for muon hodoscopes are considered. Direct calculations were performed for coordinate detector DECOR [6] and muon hodoscope URAGAN [7] which are parts of the experimental complex NEVOD (MEPhI) [8].

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2. Definitions of Coupling Functions

Coupling functions link the primary particle spectra and variations of counting rate of ground level detectors. The flux of muons at the ground level (GL) can be written as follows:

$$J_{\mu}^{GL}(E_{\mu},\theta)=\int_{E_{\mu}}^{\infty} m^{GL}(E_{\mu},E,\theta)\cdot J_{p}(E)\,dE\;, \qquad (1)$$

where $m^{GL}(E_\mu, E, \theta)$ is muon distribution function in energy E_μ with zenith angle θ from single primary proton with energy E, $J_p(E)$ – differential energy spectrum of primary protons. The detector counting rate is determined by the expression:

$$n=\int_{E_{th}}^{\infty} dE_{\mu}\int d\Omega\, J_{\mu}(E_{\mu},\theta)\cdot S(\theta,\varphi)\,, \qquad (2)$$

where $S(\theta, \varphi)$ is effective area of detector, E_{th} – its threshold energy. Here, it is assumed that muons keep the proton motion directions.

On the basis of formulas given above it is possible to define the main coupling functions. Multiplicity function - the number of muons at the surface with energy higher than E_{th} from one proton with energy E:

$$M(E,\theta)=\int_{E_{th}}^{E} m^{GL}(E_{\mu},E,\theta)\,dE_{\mu}\;. \qquad (3)$$

Yield function – the number of muons integrated over the solid angle cell with effective area $S(\theta,\varphi)$ at given direction (θ_i, φ_j):

$$P(E,\theta_i,\varphi_j)=\int_{\Delta\Omega} d\Omega\; S(\theta,\varphi)\cdot M(E,\theta)\;, \qquad (4)$$

where $\Delta\Omega$ is the region of solid angle around the direction (θ_i, φ_j). The yield function joins the differential (in primary energy) counting rate of hodoscope at the direction (θ_i, φ_j) with the primary protons flux:

$$\frac{dn(\theta_i,\varphi_j)}{dE}=P(E,\theta_i,\varphi_j)\cdot J_p(E)\,. \qquad (5)$$

The expression in the right side of (5) is the response function of hodoscope $G(E, \theta_i, \varphi_j)$. Thus, the response function is the yield function multiplied by primary proton spectrum and represents a distribution of detector counting rate at given direction in primary proton energy:

$$G(E, \theta_i, \varphi_j) = P(E, \theta_i, \varphi_j) \cdot J_p(E) \,. \qquad (6)$$

Such definitions of coupling functions allow investigate anisotropic primary CR flux variations (in this case $J_p(E)$ can depend on the angles). If the function $M(E, \theta)$ weakly changes within the solid angle range $\Delta\Omega$, the expression (4) can be re-written as:

$$P(E, \theta_i, \varphi_j) = M(E, \theta_i) \cdot \Delta S\Omega(\theta_i, \varphi_j) \,, \qquad (7)$$

where $\Delta S\Omega(\theta_i, \varphi_j)$ is the partial acceptance of the detector near the direction (θ_i, φ_j).

3. Description of Simulation Procedures

In this work, the multiplicity function $M(E, \theta)$, the yield function $P(E, \theta)$ and the response function $G(E, \theta)$ were calculated using simulation of penetration of primary protons through the atmosphere.

Simulation was performed for primary protons with energies from 1 GeV to 10 TeV at zenith angles from 0˚ to 70˚ by means of the CORSIKA code (v 6.611) [9]. In this case, fixed primary protons energies were distributed evenly in terms of the logarithm of energy with a step of 0.1 (for energies less than 20 GeV) or with a step of 0.2 (above 20 GeV). A step in zenith angle was equal to 10 degrees. For simulation of hadron interactions, combination of models SIBYLL 2.1 (for hadron energy E_h > 80 GeV) and FLUKA 2006 (for lower energies) is used.

Calculations of coupling functions were performed for coordinate detector DECOR [6] and muon hodoscope URAGAN [7]. Coordinate detector DECOR has a modular structure and consists of several 8-layer supermodules (SM) assembled on the basis of streamer tubes. Supermodules are located around the NEVOD water reservoir in the side galleries and on the NEVOD cover. For investigations of CR variations the counting rate of a special trigger which selects the events with signals from top and side SMs is used. To calculate coupling functions for DECOR, the following values of average parameters for this trigger are used: threshold energy equals to 2 GeV, the acceptance is about 1.5 $m^2 \cdot sr$ and average zenith angle is equal to 46˚.

Muon hodoscope URAGAN consists of four separate mobile supermodules with area 11.5 m^2 each, located above the NEVOD water tank. Since the URAGAN is situated in the building of experimental complex NEVOD (i.e., it is surrounded by walls and a roof), it has different threshold energies for different directions. Threshold total energies were calculated taking into account ionization energy losses of particles in the experimental building and detectors and vary from 200 MeV to 600 MeV depending on zenith angle (see Fig. 1). The effective area of URAGAN supermodule was calculated taking into account the structure of SM and also depends on zenith angle (see Fig. 2).

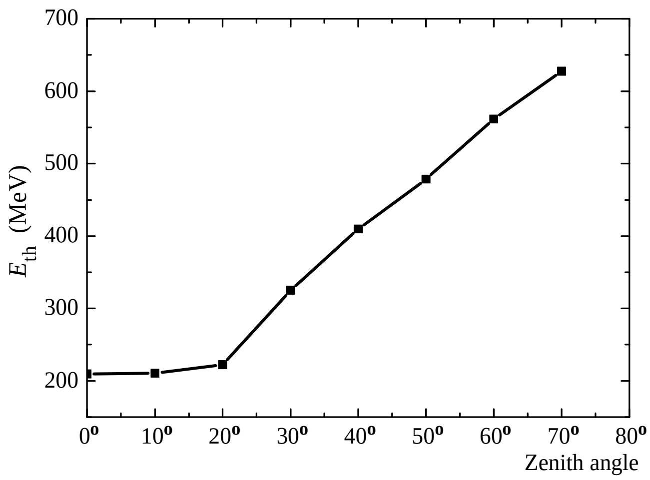

Figure 1. Threshold total energies for muon hodoscope URAGAN.

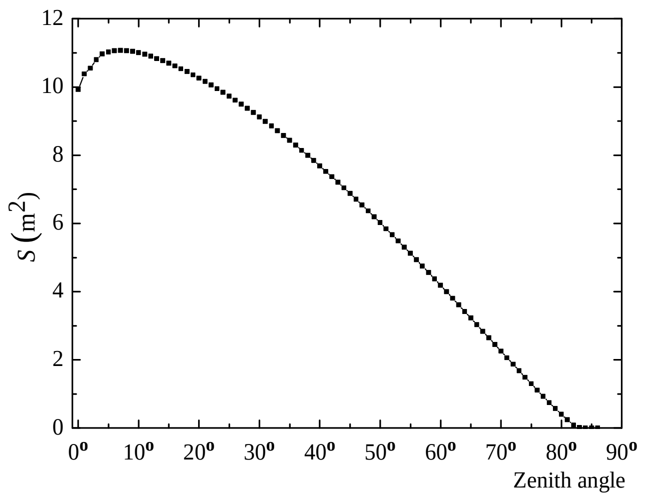

Figure 2. Effective area of a single URAGAN supermodule.

At present, three SM URAGAN are under operation, therefore in further calculations the area of one SM multiplied by 3 is used.

4. Results

In Fig. 3, the results of calculations of the multiplicity functions for the URAGAN setup and for the detector DECOR are shown. The yield functions can be calculated according to (7) if the values the partial acceptance for muon detectors at different zenith angles are used (see Table 1). To calculate the yield functions for the muon hodoscope URAGAN, the solid angle region was considered as a ring with the boundaries in zenith angle at the middles between simulation grid points.

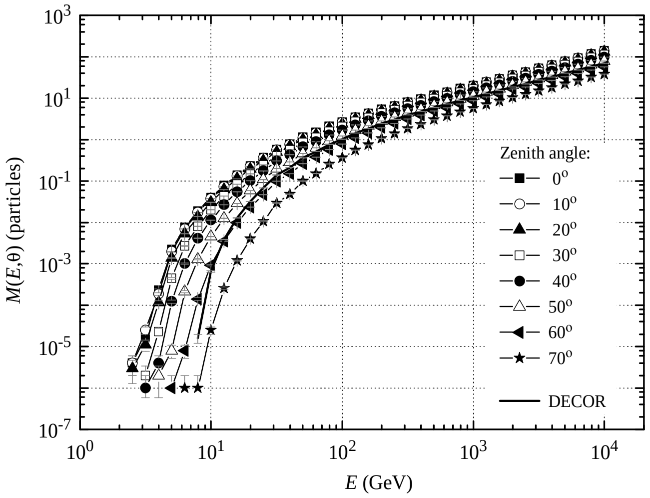

Figure 3. The dependences of multiplicity functions for URAGAN and DECOR energy thresholds on primary proton energy.

TABLE I
THE PARTIAL ACCEPTANCE OF URAGAN AND DECOR FOR DIFFERENT RANGES OF ZENITH ANGLES.

	θ °	Interval θ °	ΔSΩ, m²·sr
URAGAN	0	0-5	0.76
	10	5-15	6.1
	20	15-25	11.2
	30	25-35	14.6
	40	35-45	15.8
	50	45-55	14.7
	60	55-65	11.6
	70	65-75	6.8
DECOR	46	–	1.5

To obtain the response functions, the following CR spectra were used (energies in GeV):

1) galactic cosmic rays (GCR) [10]:

$$J_p = 1.8 \cdot E^{-2.7} \text{ [nucleons/(cm}^2\text{·s·sr·GeV)]}, \qquad (8)$$

2) solar cosmic ray (SCR) protons (values of parameters were averaged on fourteen ground level enhancement events (GLE) listed in the paper [11]):

$$J_p = 2.3 \cdot E^{-5.15} \text{ [nucleons/(cm}^2\text{·s·sr·GeV)]}, \qquad (9)$$

3) modulation of galactic CR spectrum during the Forbush decrease (FD) [12] :

$$\Delta J_p / J_p = 0.2 \cdot R^{-1}, \qquad (10)$$

where R in the rigidity in GV.

Considering the response function as the distribution of detector response in primary CR energy, it is possible to define the average energy E_{av} of primary protons which give main contribution to counting rate of the detector. For data analysis of different setups it is also convenient to use the median energy $E_{0,5}$, which divides this distribution into two equal parts, and also the lower $E_{0,05}$ and the upper $E_{0,95}$ boundaries of the energy range, below and above of which the contribution to counting rate of the detector comprises 5% (see Fig. 4). Diapason between $E_{0,05}$ and $E_{0,95}$ may be called the range of setup sensitivity in primary protons energies, which provides 90% of the setup counting rate.

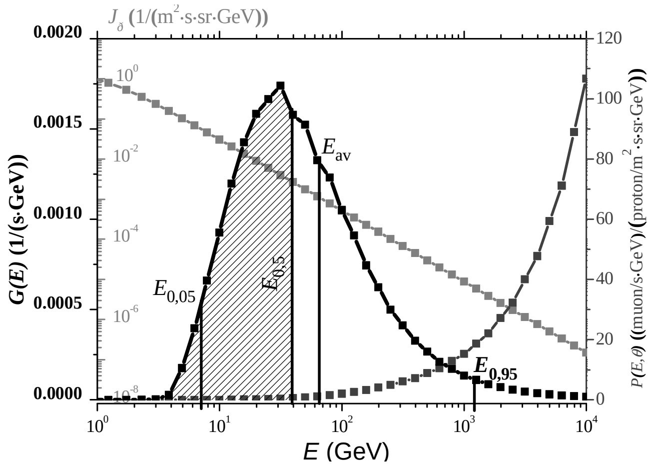

Figure 4. The response function (black curve) for galactic CR spectra and definitions of the average energy E_{av}, the median energy $E_{0,5}$, the lower $E_{0,05}$ and upper $E_{0,95}$ boundaries of energy range. Light grey curve and gray scale on the left correspond to primary CR flux. Dark grey curve (and scale on the right) represent the yield function.

The method of response functions calculations in the case of Forbush effect is different from the methods used for calculations of response functions for galactic and solar CR. This is caused by the fact that deficit of detector counting rate during Forbush effect is connected with the deficit of primary particles flux. Therefore, to calculate the setup response function for Forbush decrease the difference between response functions for usual GCR spectrum and response function for GCR spectrum during Forbush decrease, which may be obtained from (10), should be used (see Fig. 5).

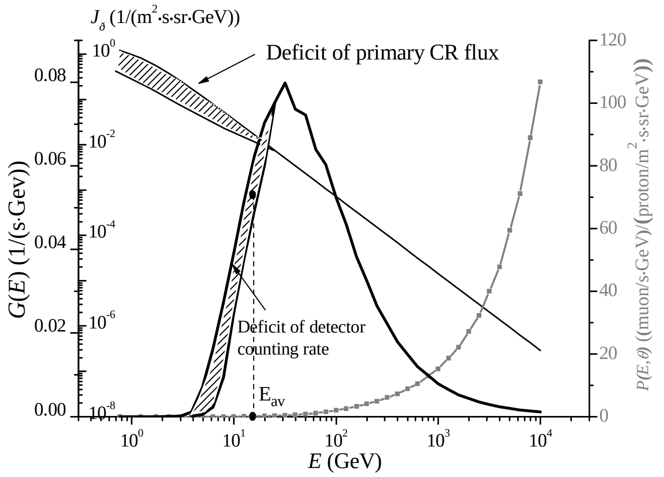

Figure 5. Definitions of the response function and average energy for the case of Forbush effect.

In Fig. 6 the dependences of average energy on the zenith angle for different forms of spectra are presented. The dependence of average energies on the zenith angle for GCR is well described by a simple formula $E_{av}(\theta) = 63 \cdot \cos^{1.08}(\theta)$ GeV.

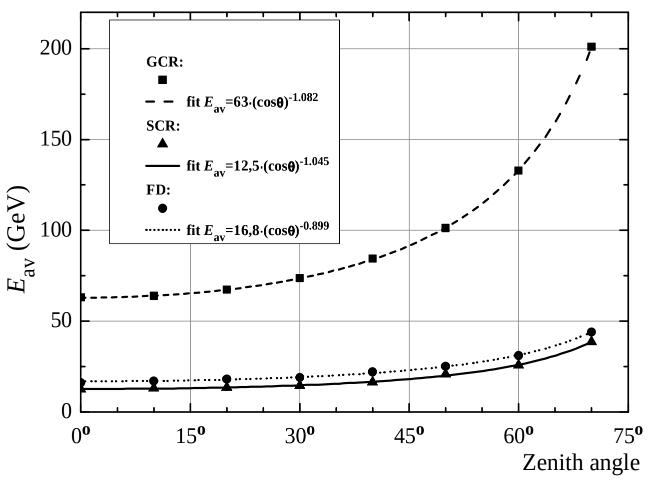

Figure 6. Dependence of average energy on the zenith angle for different forms of spectra. Symbols denote results of simulation, curves are fits by means of given formulas.

In Table 2, the lower $E_{0,05}$ and the upper $E_{0,95}$ boundaries of the energy range and median energies of primary protons for different spectra are listed.

It is seen from the table that muon detectors URAGAN and DECOR are really sensitive to the primary particle energies of tens and hundreds GeV. For example, the median primary energy for the vertical direction is equal to 38 GeV, at that 90 % of the setup response is provided by primary protons with energies from 7 to 450 GeV. For zenith angle 60°, the median energy is approximately two times higher, and the range of setup sensitivity in primary proton energies is from 14.5 to 930 GeV.

TABLE 2

THE LOWER BOUNDARIES OF THE ENERGY RANGE, THE MEDIAN ENERGIES AND THE UPPER BOUNDARIES OF THE ENERGY RANGE FOR URAGAN AND DECOR.

		θ, °	GCR	SCR	FD
$E_{0.05}$, GeV	URAGAN	0	6.8	3.3	5.1
		10	7.0	3.8	5.2
		20	7.2	4.0	5.3
		30	7.8	4.2	5.8
		40	9.2	4.3	6.7
		50	11.2	6.0	6.8
		60	14.5	7.0	8.0
		70	21.7	10.8	9.2
	DECOR	46	18	–	12
$E_{0.5}$, GeV	URAGAN	0	38	7	13
		10	39	8	14
		20	40	9	15
		30	43	10	16
		40	50	11	17
		50	59	13	20
		60	77	17	25
		70	120	26	36
	DECOR	46	82	–	28
$E_{0.95}$, GeV	URAGAN	0	450	23	52
		10	460	24	53
		20	490	25	54
		30	530	27	56
		40	590	30	58
		50	700	37	60
		60	930	50	71
		70	1380	75	86
	DECOR	46	1095	–	68

It is necessary to underline that the given values of primary proton energies relate only the GCR spectrum. Values of primary energies for SCR are considerably loss that is caused by the soft character of the spectrum. Threshold value of primary proton energies ($E_{0,05}$) in the case of SCR for the URAGAN hodoscope for the vertical direction equals to 3.3 GeV with the median energy about 7 GeV.

5. CONCLUSION

The features of the method of coupling functions calculations for muon hodoscopes are considered. On the basis of simulation with CORSIKA code, the functions of multiplicity, yield and response of detectors URAGAN and DECOR were calculated, and also average and median energies of primary protons for different types of primary spectra were obtained. Average energy of primary CR in vertical direction for muon hodoscope URAGAN is equal to 63 GeV. For DÉCOR, this value is higher and equals to 141 GeV (at θ = 46°). It is shown that the muon hodoscope in different directions is sensitive to primary flux at different energies. These results show that it is possible to evaluate changes of the primary CR in a certain energy range (from several GeV and above) on the basis of data on variations in the counting rate of a single muon hodoscope.

The results of calculations of median energies and ranges of the setup sensitivity show that in the cases of solar CR these parameters may be an order of magnitude lower than for galactic CR. This circumstance is frequently ignored in estimations of energies of solar cosmic rays detecting by muon setups during GLE events [13].

ACKNOWLEDGMENT

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