2D Stochastic Monte Carlo to evaluate the modulation of GCR for positive and negative periods

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***Abstract*— We developed a 2D model Stochastic Montecarlo for Cosmic Rays propagation in the Heliosphere. The model solves numerically the transport equation of particles in the heliosphere, including major processes affecting the heliospheric particle propagation: diffusion, convection, adiabatic energy losses and drift of particles. We evaluated the modulated flux at several distancies from the sun (i.e. at planets distance) and we compared our results for both solar polarities with measurements of CR protons at the Earth distance, 1AU.**

I. INTRODUCTION

Galactic Cosmic Rays (GCR) propagation in the heliosphere can be properly reproduced by accurate models only if all the features of the solar cavity are taken into account. We can mention for example the complex structure of the heliospheric magnetic field [1], after the measurements performed by the Ulysses satellite. The final goal of such approach is to compare the simulation results with the increasing number of observed fluxes. We used the Parker field model for the heliosphere [2], in which we included the drift effects [3]. Starting from our 1D heliospheric model that uses the stochastic simulation approach, (see [4] and [5]), we have implemented a two dimensional (radius and heliolatitude) drift model of GCR propagation in the heliosphere [6]. The 2D model becomes time dependent due to the variation of the measured values of the solar wind velocity in the ecliptic plane (V) and the tilt angle (α). This model is including curvature, gradient and current sheet drifts, which are depending on the charge sign of particles. In section III-E we present a study of the diffusion coefficients in relation to our approach, forward in time, in order to evaluate the time scale of the magnetic perturbation propagation with the solar wind. We then implemented in this model the possibility to reproduce the modulated flux at several distancies from the sun. In this way we obtain the primary CR flux at the position of the planets of the solar system. We also reproduced the GCR flux at 1 AU for different periods and compared our results with satellite data.

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II. STOCHASTIC 2D MONTE CARLO MODEL

Cosmic Rays propagation in the Heliosphere has been modeled through a stochastic Monte Carlo in two dimension (r and θ), and this is based on the Fokker-Planck equation (hereafter FPE) for GCR transport in the heliosphere without drift terms ([7] and [8]):

$$\frac{\partial f}{\partial t} = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 K_{rr}\frac{\partial f}{\partial r}\right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial\theta}\left(K_{\theta\theta}\sin\theta\frac{\partial f}{\partial\theta}\right) + \frac{1}{3r^2}\frac{\partial}{\partial r}\left(r^2 V\right)\frac{\partial}{\partial T}\left(\Gamma T f\right) - \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 V f\right) \quad (1)$$

Where θ is the heliolatitude, f is the cosmic ray number density per unit interval of particle kinetic energy, t is the time, T is the kinetic energy (per nucleon), r is the heliocentric radial distance, V the solar wind velocity and $\Gamma = (T + 2T_0)/(T + T_0)$ where T_0 is proton's rest energy. The first two terms in Eq. 1 describe the diffusion of GCR in the heliosphere, the third term is adiabatic energy loss and the last one is convection by the outgoing solar wind. Thanks to a rigorous mathematical proof by Ito [9] there is an exact equivalence between the FPE and a set of stochastic differential equations (SDE), so we can apply this technique to our numerical simulation solving ordinary differential equations. If we consider the coordinates variation in a small time step Δt for a test quasi-particle, the SDE equivalent to eq. 1 can be written as:

$$\Delta r = \frac{1}{r^2}\frac{d(r^2 K_{rr})}{dr}\Delta t + V\Delta t + R_g\sqrt{2K_{rr}\Delta t}$$

$$\Delta\mu = \frac{d}{d\mu}\left[\left(1-\mu^2\frac{K_{\theta\theta}}{r^2}\right)\right]\Delta t + R_g\sqrt{2\left(1-\mu^2\right)\frac{K_{\theta\theta}}{r^2}\Delta t}$$

$$\Delta E = -\frac{2VT\Gamma}{3r}\Delta t \quad (2)$$

where $\mu = cos\theta$, so $\Delta\mu = \Delta cos\theta$ is the latitudinal variation of the particle, V is solar wind velocity, R_g is a Gaussian distributed random number with unit variance, and Δt is the time step of calculation. The radial diffusion coefficient is $K_{rr} = K_{||}cos^2\psi + K_{\perp}sin^2\psi$, where ψ is the angle between radial and magnetic field directions [7]. The latitudinal coefficient is $K_{\theta\theta} = K_{\perp}$. The parallel and the perpendicular diffusion coefficients ([10]) are

$$K_{||} = K_0 \beta K_P(P) \frac{B_{\oplus}}{3B}$$
$$K_{\perp} = (K_{\perp})_0 K_{||} \tag{3}$$

where $K_0 = 1 - 6 \times 10^{22}$ cm^{2}s^{-1} (or in unit of au^{2}s^{-1} becomes $K_0 \simeq 0.5 - 3 \times 10^{-4}$), β is the particle velocity, P is the CR particle's rigidity, $K_P(P)$ take into account the dependence on rigidity (in GV), $(K_{\perp})_0$ is the ratio between parallel and perpendicular diffusion coefficient, $B_{\oplus} \sim 5$ nT is the value of heliospheric magnetic field at the Earth orbit, and B is the Parker field magnitude [11]. In our model the solar wind speed $V(\theta)$ is a function of the heliolatitude θ [12]: $V = V_0(1 + cos\theta)$, for $0^o < \theta < 60^o$ and $V = 750$ kms^{-1}, for $60^o < \theta < 90^o$. V_0 is the velocity of solar wind in ecliptic plane. We used the Parker model for the heliospheric magnetic field because this allows an analytical solution for drift velocities and better evaluation of diffusion tensor. Drift effects are included through analytical effective drift velocities: in this spiral field we added to the previous formulas (2), the three components of drift (gradient and curvature drift plus drift along the neutral sheet, [13]) to calculate the position of a test particle during a time step Δt:

$$\Delta r_d = \Delta r + (v_g + \overline{v_d^{ns}})\Delta t$$
$$\Delta\mu_d = cos\left[\arccos(\Delta\mu) + \arctan\left(\frac{v_\theta \Delta t}{r}\right)\right] \tag{4}$$

Δr_d is the radial variation with drift effect, $\Delta\mu_d$ is the latitudinal variation of the particle due to drift, v_g is the velocity of gradient drift, $\overline{v_d^{ns}}$ is the velocity of neutral sheet drift and v_θ is the velocity of curvature drift. The average drift velocity is :

$$v_d = \frac{pv}{3q}\nabla \times \frac{\vec{B}}{B^2} = \nabla \times (\frac{\beta P}{3B}) \tag{5}$$

We adopted the matematical solution from Hatting and Burger [15], the so-called wavy neutral sheet (WNS) model, for steady-state drift dominated modulation (our case). In this model the average of the drift velocity (5) is taken over one solar rotation. The drift velocity, that remains divergence-free, becomes:

$$v_d = g(\theta)\nabla \times (\frac{\beta P}{3B}) +$$
$$\frac{v}{6}\frac{2pr\Gamma'}{qA(1+\Gamma'^2)(\alpha + \Delta\theta_{ns})}\vec{e}_r \tag{6}$$

with

$$g(\theta) = 1$$
$$\text{if} \quad 0 \le \theta \le \frac{\pi}{2} - \alpha - \Delta\theta_{\rm ns}$$
$$g(\theta) = \frac{2}{\pi}\sin^{-1}(\frac{\frac{\pi}{2} - \theta}{\alpha + \Delta\theta_{ns}})$$
$$\text{if} \quad \frac{\pi}{2} - \alpha - \Delta\theta_{\rm ns} < \theta < \frac{\pi}{2} + \alpha + \Delta\theta_{\rm ns}$$
$$g(\theta) = -1$$
$$\text{if} \quad \frac{\pi}{2} + \alpha + \Delta\theta_{\rm ns} \le \theta \le \pi \tag{7}$$

where $\Gamma' = \frac{\Omega}{V}(r - r)\cos(\alpha/\sqrt{2})$. The second term of equation (6), the neutral sheet drift, is equal to zero outside the neutral sheet region (in the 2D model this region is wide as the tilt angle). The function $g(\theta)$ has the effect of scaling the curvature and gradient drift (first term) down over the neutral sheet region so that it is zero at $\theta = \pi/2$.

As Local Interstellar Spectrum of protons (LIS) we used Burger's model [14].

III. Study of the Modulation parameters

A. *Diffusion coefficient*

We evaluated the dependence of the modulation effect from the diffusion coefficient value, in the quasi linear theory approximation (i.e. $K_P = P$). In the 2D version the analysis of the relation between the diffusion coefficient and the modulation strenght becomes more complex. The modulation as a function of the diffusion coefficient follows the expected behaviour, so higher values of K_0 corresponds to a lower modulation and vice versa (see Fig. 1 for CR spectra at Earth with different values of K_0). We evaluated also the dependence of our model on the ratio between parallel and perpendicular diffusion coefficient, over the commonly used range, $(K_{\perp})_0 = 0.01 - 0.05$. The modulation effect on the LIS is higher for lower values of $(K_{\perp})_0$, these results are not shown here, just the expected and best value for $A > 0$ period are used, as reported in IV-A.

B. *Tilt angle α and Solar Wind velocity V_{sw}*

For positive periods we have evaluated the effect on the model by changing the solar wind speed in the range $V_0 = 100 - 1000$ kms^{-1} and tilt angle in the range $\alpha = 10^o - 50^o$. We used (see [6] for a complete description) the tilt angle α as main parameter for the level of the solar activity: the higher the value of α the lower the expected GCR flux, for both solar field polarities. Besides for the same value of α a higher flux of protons is expected for $A > 0$. We also evaluated the effect of ecliptic solar wind velocity V_0 (a higher value of V_0 means higher solar activity).

C. *Drift effects*

To study the effect of the drift velocity terms we considered a period when $\alpha = 30^o$ and $V = 400$ kms^{-1} for $A > 0$ (see [16]) and a period with $A < 0$ (see [17]). After computing

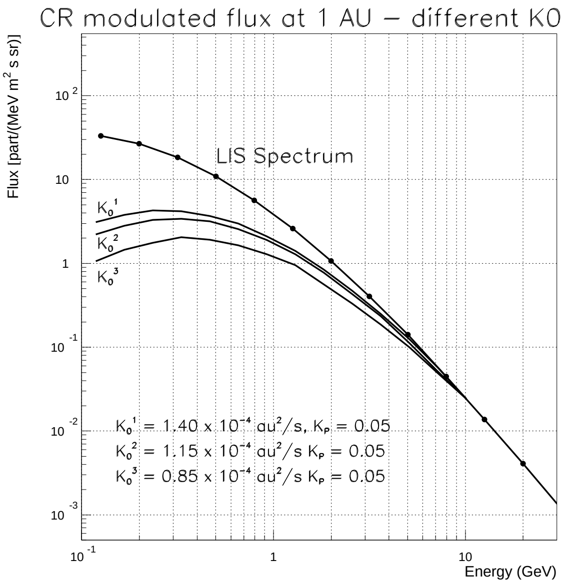

Fig. 1. 2D model:

the flux including all the terms, we have excluded the drift from the transport equation. The drift effect seems to be more evident in positive periods. We have studied the three main drift components in the heliosphere: gradient, curvature and neutral sheet (NS) drifts, and as each one of the drift terms affects the modulated spectrum and it seems that neutral sheet is the dominant one in what is called the neutral sheet region, (see Eq. 6 and 7) for both solar polarities but stronger for $A > 0$ (see [16]).

D. Data set

We choosed CR proton data from 4 different experiments in order to compare and tune our model results. These are: AMS-01 ([18]), IMAX ([19]), Caprice ([20]) and BESS ([21]). First three experiments took data in a period of positive solar polarity, while last one is in negative solar period. The corresponding periods for measured proton flux are: june 1998 (AMS), july 1992 (IMAX), august 1994 (Caprice) and august 2002 (BESS). Solar wind values for selected periods were obtained from omniweb ([22]) choosing 27 days average, while tilt angles Wilcox Solar Laboratory ([23]) as reported in Table I.

TABLE I
EXPERIMENTS PARAMETERS I

Experiment	Tilt angle α	Solar wind velocity V_{sw}
AMS-01	30°-45°	430 km/s
IMAX	20°-40°	400 km/s
Caprice	10°-25°	440 km/s
BESS	35°-50°	420 km/s

Diffusion coefficients are needed by our model to evaluate the proton CR modulation in different conditions, so we get them from [24] and [25], (see Table II). These values are based on long term study of neutron monitors measurements and the Force Field model approach to the Heliosphere (see [26]).

TABLE II
EXPERIMENTS PARAMETERS II

Experiment	Diffusion coefficient (au²/s) K_0	$(K_\perp)_0$
AMS-01	1.70×10^{-4}	0.025
IMAX	1.33×10^{-4}	0.025
Caprice	1.90×10^{-4}	0.025
BESS	0.88×10^{-4}	0.05

E. Average parameters

Our model has been realized with a specific structure in order to reproduce the real behaviour of CR entering the heliosphere from its outer limit, the heliopause, located approximately (see [27]) at 100 AU, and travelling down to the solar system planets and the Earth at 1 AU. Besides we use fixed values of tilt angle α, solar wind V_{sw} and diffusion coefficient K_0. We started looking carefully if this was a good approximation to reproduce the solar modulation of CR. First we evaluated the time t_{sw} needed by the Solar Wind (a plasma that carries the Sun magnetic field) to expand from the outer corona up to the heliopause. With SW velocities ranging from 200 km/s to 600 km/s it takes something between 2.4 and 0.8 years. On the contrary the period τ_{ev} of the stochastic evolution of a quasi particle inside the heliosphere from 100 AU down to 1 AU is between a little more than 1 month (200 MeV) and few days (10 GeV). This scenario, where $\tau_{ev} < t_{sw}$, clearly shows that fied parameters are no longer useful for such a study. In fact if for example we are interested in the CR modulation of a period t_X at 1 AU, this is related to a different condition at 100 AU where particles are injected. If we consider in a first approximation τ_{ev} negligible with respect to t_{sw} (for 400 km/s this time is a little more than 1 year) we can say that at 100 AU the conditions are those present at Earth almost 1 year before. To take into account this, we first decided to move from fixed values of α, V_{sw}, K_0, to average values of these quantities. The first approach was two different kind of average values for all quantities: first one was the average over 12 months centred in the month we are considering (when the corresponding detector was collecting data), so α^{cent}, V_{sw}^{cent}, K_0^{cent}, the second one was the average 12 months back in the past, α^{back}, V_{sw}^{back}, K_0^{back} (see Table III and IV).

TABLE III
EXPERIMENTS PARAMETERS IA

Experiment	) K_0^{cent}(au²/s	$(K_\perp)_0$	α^{cent}	V_{sw}^{cent}
AMS-01	1.80×10^{-4}	0.025	26° - 45°	400 km/s
IMAX	1.25×10^{-4}	0.025	20° - 36°	440 km/s
Caprice	2.07×10^{-4}	0.025	12° - 27°	500 km/s
BESS	1.07×10^{-4}	0.05	45° - 58°	450 km/s

TABLE IV

EXPERIMENTS PARAMETERS IB

Experiment	K_0^{back}(au^2/s)	$(K_\perp)_0$	α^{back}	V_{sw}^{back}
AMS-01	1.87×10^{-4}	0.025	13° - 28°	380 km/s
IMAX	1.01×10^{-4}	0.025	32° - 50°	460 km/s
Caprice	2.01×10^{-4}	0.025	16° - 33°	510 km/s
BESS	1.04×10^{-4}	0.05	40° - 55°	420 km/s

TABLE V

EXPERIMENTS PARAMETERS II

Experiment	K_0^{back}(au^2/s)	$(K_\perp)_0$	α^{back}	V_{sw}^{back}
AMS-01	1.46×10^{-4}	0.025	50°	430 km/s
IMAX	1.20×10^{-4}	0.025	55°	410 km/s
Caprice	1.40×10^{-4}	0.025	30°	500 km/s
BESS	0.60×10^{-4}	0.05	40°	500 km/s

IV. RESULTS

A. Comparison with data

We started our simulations form the values in Table I and Table II choosing $(K_\perp)_0 = 0.025$ for the period with $A > 0$ (as reported for example in [28]) and $(K_\perp)_0 = 0.05$ for $A < 0$. Results of these simulation are shown in Figures 2, 3, 4 and 5, and it can be clearly seen that simulated fluxes are higher than measured ones. This fact, together with all statements in Sec. III-E, convinced us to try a more realistic approach, so we used parameters averaged through a large enough period (12 months), to keep into account the expansion of magnetic disturbancies driven by the expanding solar wind. So we started from average values as reported in Table III and IV and we performed a fine tuning of parameters in order to reproduce the measured fluxes. In Table V we report the best values for solar parameters in order to have a good fit with data, an fluxes can be seen in Fig. 6,7 and 8. First of all we must notice that best fit value differ from average parameters, the only good agreement is for IMAX (1.20×10^{-4} compared with $1.01 - 1.25 \times 10^{-4}$), while for AMS and Caprice best fit values are far away from average values. Negative period (BESS) play a different role in this quest: average values, especially those back in time that seem to be closer to the real behaviour of the heliosphere, probably are too close to the inversion of magnetic field (approx. at the beginning of year 2001). We are still investigating this fact and the effective difference of $(K_\perp)_0$ values between the two polarities.

B. CR flux modulation for solar system planets

We present our results for the modulated CR spectrum at different heliocentric distances. In particular we focused on the solar system planets, some of which have been already interested in space missions, the others will be investigated by robots or human missions in the near future (see Table VI for the solar distancies). In Fig. 9 we show the CR flux for the external Solar System planets, Neptune, Uranus and Pluto, in the same conditions obtained for the Caprice experiment

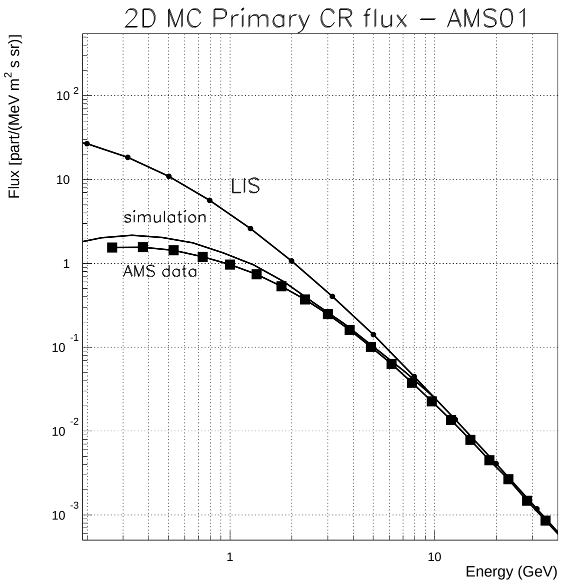

Fig. 2. 2D model: CR modulation flux at Earth, AMS-01 estimated parameters from Table I and II

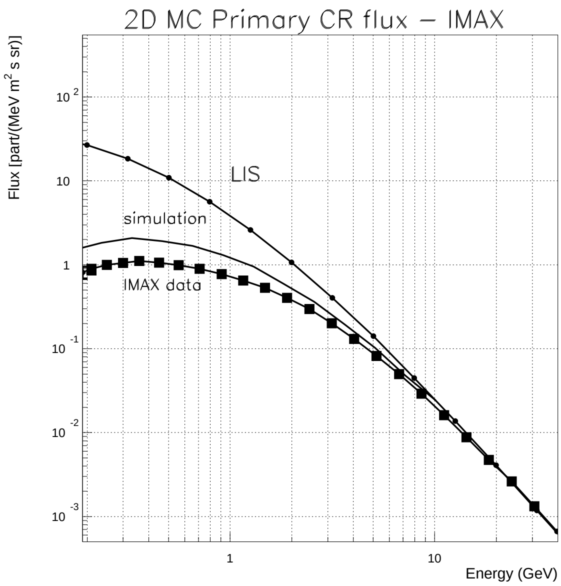

Fig. 3. 2D model: CR modulation flux at Earth, IMAX estimated parameters from Table I and II

(see Sec. IV-A). As shown the solar modulation is decreasing with increasing heliospheric distance (this is in agreement with integral spectrum of CR for example from Voyager I and II and Pioneer, see [29]).

V. CONCLUSIONS

We built a 2D (radius and heliolatitude) stochastic model of particles propagation across the heliosphere starting from a 1D model (only radius). Our model takes into account drift effects

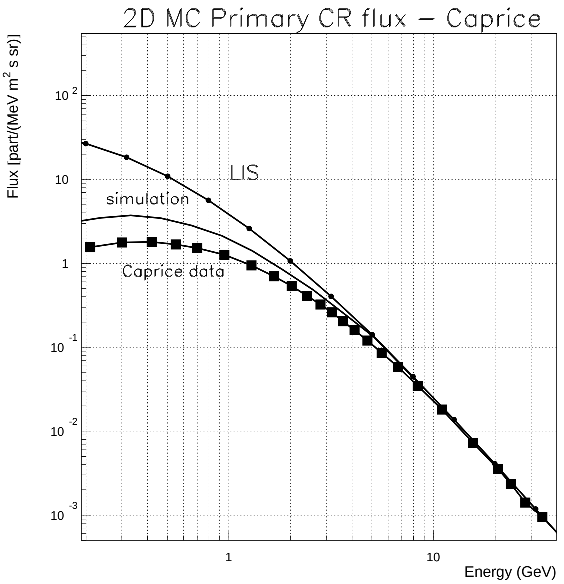

Fig. 4. 2D model: CR modulation flux at Earth, Caprice estimated parameters from Table I and II

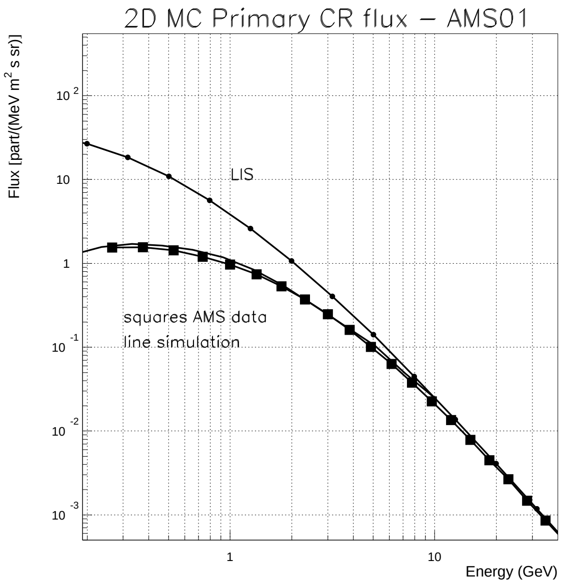

Fig. 6. 2D model: CR modulation flux at Earth, AMS-01 best parameters from Table V

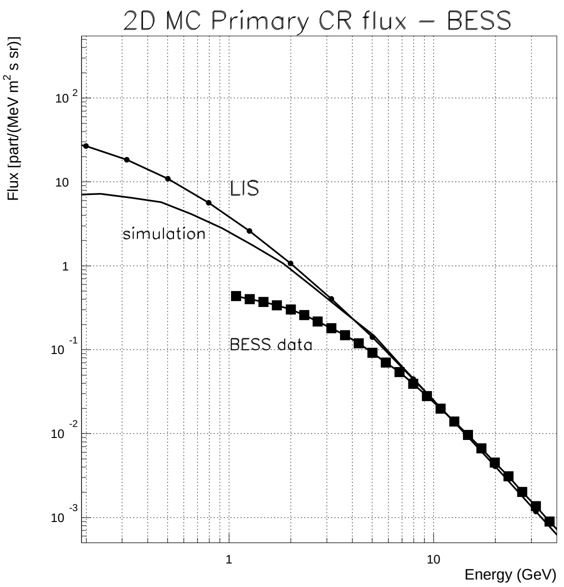

Fig. 5. 2D model: CR modulation flux at Earth, BESS estimated parameters from Table I and II

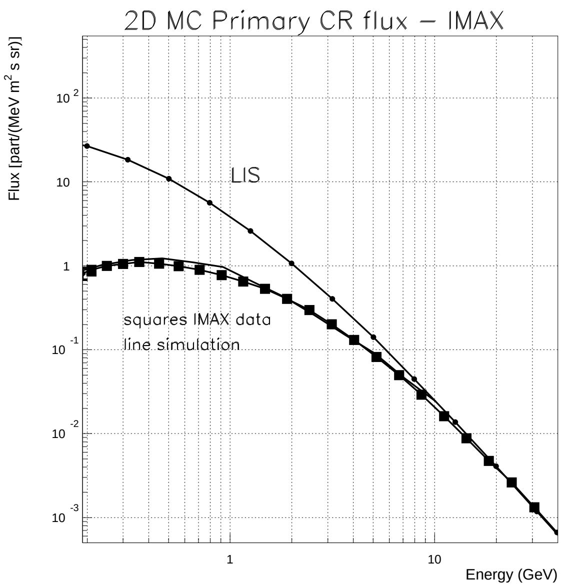

Fig. 7. 2D model: CR modulation flux at Earth, IMAX best parameters from Table I and II

and show quantitatively good agreement with measured values. Proton spectra, as predicted by the model, are decreasing with increasing tilt angles and solar wind velocity. We investigated the effect of the different drift componets, in particular the neutral sheet drift. For positive periods as expected the effect of this drift component is to increase the observable flux at 1 AU (Earth). We compared our simulations with measured data from 4 different experiments, AMS, IMAX, Caprice and BESS. Starting from fixed values for the main parameters (K_0, α and V_{sw}) for the related period, we moved to averaged values of the same parameters back in the past, in order to reproduce the propagation of incoming CR through magnetic disturbancies carried by the outgoing solar wind. This approach seems to be better at a first approximation and will be investigated deeply in relation to solar conditions and particle energy. The model must be tested with data in different solar conditions (in particular $A < 0$), we thus expect the next AMS-02 mission for the long data taking period, more than 3

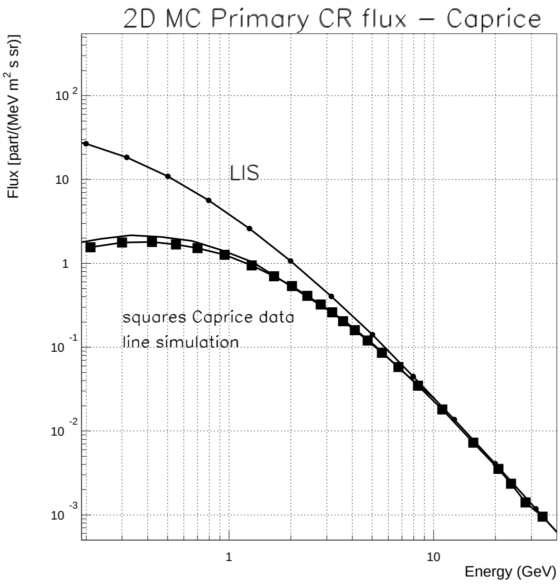

Fig. 8. 2D model: CR modulation flux at Earth, Caprice best parameters from Table I and II

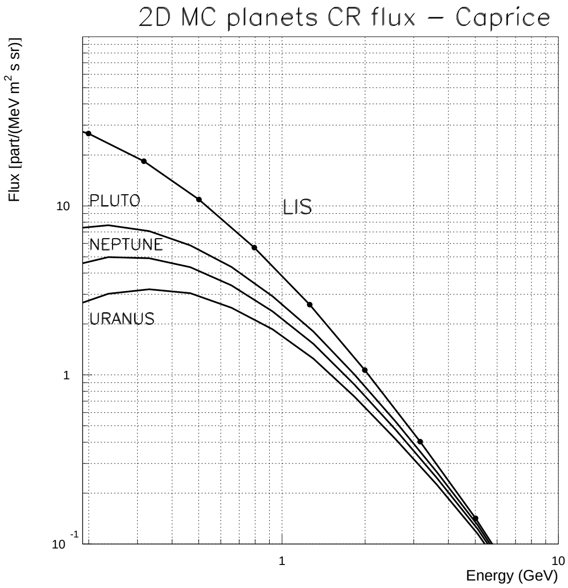

Fig. 9. Modulated CR flux for 3 external planets of the Solar System

TABLE VI

SOLAR SYSTEM PLANETS

Planet	Distance (in AU) from the Sun	Space Missions
Mercury	0.387	BepiColombo
Venus	0.723	Venus Express
Earth	1.0	
Mars	1.524	Mars Express
Jupiter	5.203	Ulysses
Saturn	9.539	Cassini-Huygens
Uranus	19.18	
Neptune	30.06	
Pluto	39.44	

years, in ascending solar phase and negative polarity.

Finally we implemented in the 2D model the possibility to calculate a modulated proton spectra at different heliospheric distancies (corrispondent to the solar system planets radii).

REFERENCES

[1] L.A. Fisk, *J. Geophys. Res.* 101, 15547-15553, 1996.
[2] R. A. Burger and M. Hitge,*American Geophysical Union* SH71A-04, Fall Meeting 2002.
[3] G. Wibberenz, et al., *Space Science Reviews*, 97, 373 2001.
[4] M. Gervasi et al., *Nuclear Phys. B* (proc. suppl.) 78, 26 1999.
[5] M. Gervasi, P.G. Rancoita and I. Usoskin, “Transport of Galactic Cosmic Rays in the Heliosphere: Stochastic Simulation Approach”, *Proceedings of the 26th ICRC* SH 3.1.18, 69 1999.
[6] P. Bobik, M. Gervasi, D. Grandi, P.G. Rancoita and I. Usoskin, “2D stochastic simulation model of cosmic ray modulation: comparison with experimental data”, *Proceedings of ICSC*, 2003ESA SP-533, 2003.
[7] M. S. Potgieter et al., *Astrophys. J.*, Part 1 (ISSN 0004-637X) 403, no. 2, 760 1993.
[8] L. A. Fisk, *J. Geophys. Res.* 81, 4646, 1976.
[9] Gardiner, C.W., *Handbook of Stochastic Methods*, Springer Verlag, berlin, 1989.
[10] M. S. Potgieter, J. A. Le Roux, *Astrophysical Journal* 423, 817, 1994.
[11] R.A. Burger and M.S. Potgieter, *The Astrophys. J.* 339, 501, 1989.
[12] J. P. L. Reinecke, C. D. Steenberg, H. Moraal, F. B. McDonald, *Advances in Space Research* 19, Issue 6, 901, 1997.
[13] M. Hatting and R.A. Burger, *Adv. Space. Res.* 16, No. 9, 213, 1995.
[14] R. A. Burger, M. S. Potgieter, B. Heber, *J. Geophys. Res. 105*, Issue A12, 27447, 2000.
[15] R. A. Burger and M. Hatting, *Astr. and Space Sc.*, 230, 375-382, 1995.
[16] P.Bobik, M. J. Boschini, D. Grandi, M. Gervasi, and P. G. Rancoita “A stochastic Montecarlo approach to Solar modulation of GCR: evaluation of the proton flux at several distances from the Sun”, *Proceedings of the 10th ICATPP Conference*, World Scientific, 2008.
[17] P.Bobik, M. J. Boschini, D. Grandi, M. Gervasi, and P. G. Rancoita “A 2d stochatsic Montecarlo for the Solar Modulation of GCR: a procedure to fit Interplanetary Parameters comparing to the experimental data”, *Proceedings of the 9th ICATPP Conference*, 206-211, World Scientific, 2006.
[18] J. Alcaraz et.al., *Physics Letters B* 490, 27 (2000).
[19] L.M. Barbier et al. *The Astrophysical Journal* 533, 281, 2000.
[20] M. Boezio et al. *The Astrophysical Journal* 518, 457, 1999.
[21] S. Haino et al. *Phys. Lett. B* 594, 35, 2004.
[22] http://omniweb.gsfc.nasa.gov/form/dx1.html
[23] http://quake.stanford.edu/∼wso/wso.html
[24] Moskalenko et al., *The Astrophysical Journal* 565, 280, 2002.
[25] Usoskin et al., *J. Geophys. Res.* 110, A12108, 2005.
[26] Gleeson L. J., and Axford W. J., *Astrophys. J.*, 154, 1011, 1968.
[27] Decker, R. B. et al. *American Geophysical Union, Fall Meeting 2007*, 2007.
[28] J. Giacalone, J. R. Jokipii, *Astrophysical Journal* 520, Issue 1, 204 1999.
[29] http://pds-rings.seti.org/voyager/