The iteration method to obtain the analytical solutions of the boundary problems for cosmic ray propagation theory

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Abstract. **In this paper the iteration method of solution of the cosmic rays (CR) propagation in space homogeneous interplanetary scattering medium problems is proposed. The method is based on smallness of the anisotropy CR degree. The comparison of exact analytical solution and iteration solution for the constant diffusion coefficient was done. The iteration solutions for different values of the diffusion coefficient depending on energy are obtained.**

1. INTRODUCTION

It is very important to obtain the exact analytical solutions of galactic cosmic ray propagation problems in order to study "cosmic weather" and other astrophysical phenomena in heliosphere. But exact solutions exist only for some rare cases. Therefore it is necessary to develop another methods of these solutions.

2. COMPARISON OF ITERATIVE SOLUTIONS WITH THE EXACT ANALYTICAL SOLUTION

We use the iteration method considered in the work [1]. The following transport equation in form [2,4] for the stationary case is used:

$$div\vec{j} = -div_p j_p, \tag{1}$$

where $\vec{j} = -\chi \frac{\partial N}{\partial \vec{r}} - \frac{p\vec{u}}{3}\frac{\partial N}{\partial p}$, – CR flow; $j_p = \frac{\vec{u}p}{3}\frac{\partial N}{\partial \vec{r}}$, -is flow in momentum space, χ - coefficient of diffusion, $\vec{u}$ - velocity of a solar wind, p - module of a particle in impulse space. To apply the iteration method the equation (1) may be written in form:

$$div\vec{j}_n = -div_p(j_p)_{n-1} \tag{2}$$

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The zero approximation occurs as a solution of equation:

$$\vec{j}_0 = -\chi\frac{\partial N_0}{\partial \vec{r}} - \frac{p\vec{u}}{3}\frac{\partial N_0}{\partial p}, \tag{3}$$

under the condition $N_0(r_0, p) = N_b(p)$, where $N_b(p)$ is spectrum of CR outside the Solar wind. It is well known force field solution [3]. The first approximation is found from equation (3), and the second aproximation will depend on the first respectively:

$div\vec{j} = -div_p j_p$ and the same way to obtain another approximation.

Then general solution may be written as a series expansion: $N(r,p) = \sum_{n=0}^{\infty} N_n$. Boundary value of power's spectrum with exponent $-\gamma = 2.5$ is defined. In terms of non-dimensional momentum $\eta = \frac{p}{m_0 c}$, this spectrum is represented as following: $N_b = \eta^{-1}(1+\eta^2)^{-\frac{\gamma+1}{2}}$. After some transformations we obtain iterative solutions for the following cases:

1) $\chi = const$

$$N_1 = N_0(\eta e^{-\frac{x-x_0}{3}})(x_0 - x) + \frac{2}{x_0}\int_0^{x_0} \xi(\frac{\partial N_0}{\partial \xi}(\eta e^{-\frac{x-x_0}{3}-\frac{\xi-x_0}{3}}) -$$
$$N_0(\eta e^{-\frac{x-x_0}{3}-\frac{\xi-x_0}{3}}))d\xi - \frac{2}{x}\int_0^{x} \xi(\frac{\partial N_0}{\partial \xi}(\eta e^{-\frac{\xi-x_0}{3}}) - N_0(\eta e^{-\frac{\xi-x_0}{3}}))d\xi -$$
$$-2\int_x^{x_0} \frac{1}{\psi}d\psi \int_0^{\psi} \xi(\frac{\partial^2 N_0}{\partial \xi \partial \psi}(\eta e^{-\frac{x-\psi}{3}-\frac{\xi-x_0}{3}}) - \frac{\partial N_0}{\partial \psi}(\eta e^{-\frac{x-\psi}{3}-\frac{\xi-x_0}{3}}))d\xi,$$

$$N_0 = \eta^{-1} e^{-\frac{x-x_0}{3}}(1+\eta^2 e^{-\frac{2}{3}(x-x_0)})^{-\frac{\gamma+1}{2}},$$

where $x = \frac{ur}{\chi}, x_0 = \frac{ur_0}{\chi}, N_0, N_1$ – first and second allowance respectively.

2) $\chi \square p$

$$N_1 = \frac{N_0(\eta - \frac{x-x_0}{3})(x-x_0)}{\eta} - 2N_0(\eta - \frac{x-x_0}{3})\ln(\frac{x_0}{x}) +$$
$$+\int_x^{x_0} \frac{2}{\psi^2} d\psi \int_0^{\psi} N_0(\eta - \frac{x+\xi-\psi-x_0}{3})(1+\frac{\xi}{\eta})d\xi,$$

$$N_0 = (\eta - \frac{x-x_0}{3})^{-1}(1+(\eta - \frac{x-x_0}{3})^2)^{-\frac{\gamma+1}{2}}$$

3) $\chi \square p^2$

$$N_1 = \frac{N_0(\eta^2 - \frac{2}{3}(x-x_0))}{\eta}(x-x_0) - 2\eta N_0(\eta^2 - \frac{2}{3}(x-x_0))\ln(\frac{x_0}{x})$$
$$+\int_x^{x_0} \frac{2}{\psi^2} d\psi \int_0^{\psi} N_0(\eta^2 - \frac{2}{3}(x+\xi-\psi-x_0))(\eta+\frac{\xi}{\eta})d\xi,$$
$$N_0 = (\eta^2 - \frac{2}{3}(x-x_0))^{-\frac{1}{2}}(1+\eta^2 - \frac{2}{3}(x-x_0))^{-\frac{\gamma+1}{2}}$$

The efficiency of the method was verified by comparison of its solution with the exact analytical solution of cosmic ray propagation problem in interplanetary medium for the case when $\chi = const$ by means of Melin transformations. As a result, for $\chi = const$, exact solution is presented as follows [2]:

for $\eta \geq 1$:

$$N = \sum_{n=0}^{\infty} C^{n}_{-\frac{\gamma+1}{2}} \cdot \eta^{-\gamma-2-2n} \cdot \frac{F(\frac{2}{3}(\gamma+2n+2),2,x)}{F(\frac{2}{3}(\gamma+2n+2),2,x_0)}$$

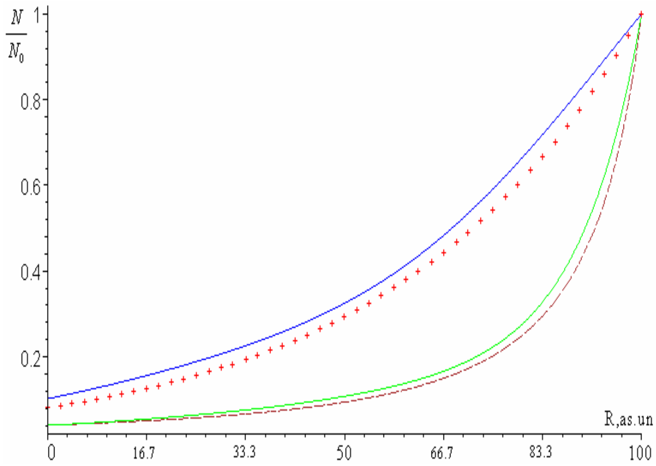
for $0 \leq \eta < 1$:

$$N = \sum_{n=0}^{\infty} C^{n}_{-\frac{\gamma+1}{2}} \cdot \eta^{2n-1} \frac{F(\frac{2}{3}(-2n+1),2,x)}{F(\frac{2}{3}(-2n+1),2,x_0)} +$$
$$+\frac{3}{4}\sum_{n=0}^{\infty} \eta^{\frac{3\alpha_n}{2}} \cdot \frac{\Gamma(\frac{\gamma+3\alpha_n/2}{2}+1)\cdot\Gamma(-\frac{3\alpha_n}{4}-1/2)}{\Gamma(\frac{\gamma+1}{2})} \cdot \frac{F(-\alpha_n,2,x)}{F'_{\alpha}(-\alpha_n,2,x_0)}$$

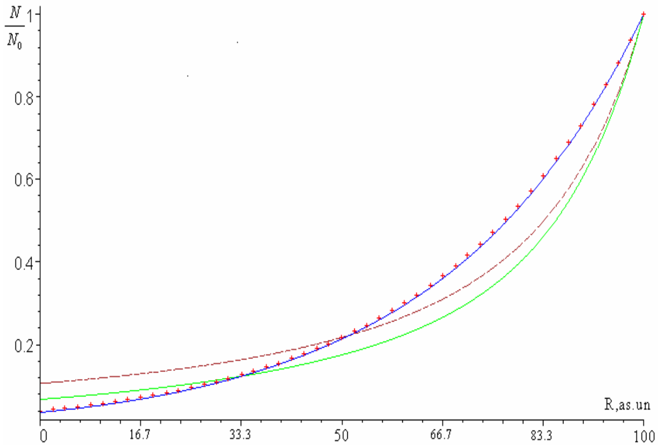
where α_n are the roots of the equation : $F(-\alpha_n,2,3)=0$, where $F(a,b,x)$ is confluent hyperheometric function.

Comparison of iteration solutions with the exact analytical solution are illustrated in Fig.1

for $\eta = 0.5$



for $\eta = 1$



for $\eta = 10$

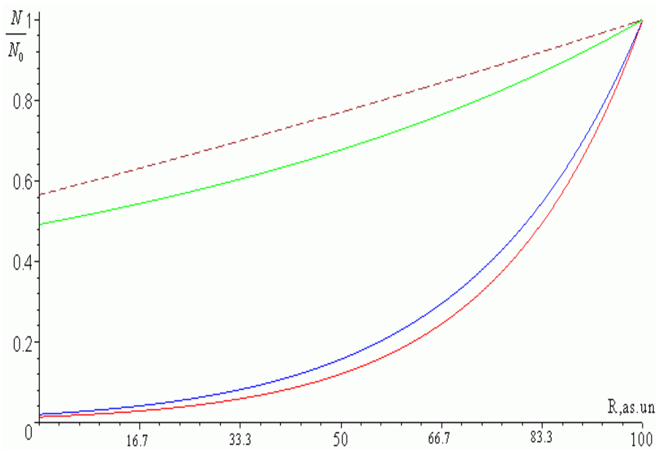
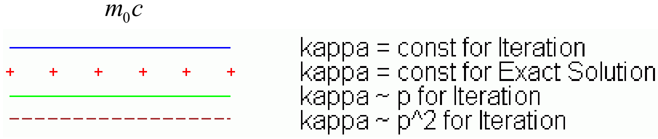


Fig 1. Dependency of CR concentration is normalized concentration on the boundary of heliosphere $(\frac{N}{N_0})$ depending on the heliosphere distance R (for values $\frac{p}{m_0 c} = 0.5;1;10$)



3. CONCLUSION

As it was seen, CR concentration that was obtained by means of two step iteration method does not differ significantly from the analytical solution. Therefore iterative solutions developed by this method can be used for the cases when $\chi = p$, $\chi = p^2$ and for the numerous complicated situations where exact analytical solutions can not be received in principle.

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