Stochastic acceleration of cosmic rays in helical plasma turbulence

Yurij Fedorov[1], Milan Stehlik[2]

Abstract— **The acceleration of energetic particles is investigated on the base of kinetic equation for cosmic ray distribution function in a statistically anisotropic turbulent magnetic field. The cosmic ray acceleration by the turbulent field is considered using the diffusion approach. The diffusion approximation of the kinetic equation is examined for particle distribution function which is nearly isotropic. It is known that large-scale magnetic and electric field are generated in a medium with statistically anisotropic magnetohydrodynamical turbulence. The particle acceleration by large scale electric field can be very efficient if the magnetic helicity of turbulent medium is sufficiently high. The effectiveness of this acceleration mechanism is compared with Fermi acceleration of the second order.**

I. INTRODUCTION

Theory of charged particle transport in magnetohydrodynamic (MHD) turbulence still attracts large attention due to many applications in both laboratory and astrophysical plasma. Despite of enormous effort, numerous problems concerning particle acceleration wait for their resolutions and the related problems and questions remains deficiently explored. Besides a lot of other questions concerning the creating and/or amplifying of magnetic fields (MF) by turbulence (see in [20], [14]), another question is, how does MHD turbulence influence to transport of heat and cosmic rays [4], [2]. MHD turbulence is an important agent for particle acceleration as was pointed first by Fermi [9].

The second-order Fermi mechanism (stochastic acceleration of particles by scattering with randomly moving magnetized clouds) has application in a wide range of astrophysical objects including the solar wind (SW) and solar flares [19], [1], cluster of galaxies [5], the Galactic center [15], etc. Usually, this mechanism was applied in cases, where protons are accelerated from a thermal distribution.

Another relating stochastic acceleration mechanism is connected with the created magnetic field in the helical MHD turbulence [7], [10], [22]. In fact, due to an anisotropic helical turbulence the large scale electric field is generated [14], [27], effect known as α-effect. The electric field is directed along regular magnetic field (MF) and it can efficiently accelerate charged particles in the turbulent plasma. The equation describing such acceleration mechanism (α-acceleration) was firstly derived in [12]. The stochastic particle acceleration by anisotropic helical turbulence was investigated on the frame of kinetic equation for charged energetic particle distribution function in the paper [7].

[1]MAO NASU Kiev, Ukraine, fedorov@mao.kiev.ua
[2]IEP SAS Košice, Slovakia, stehlik@saske.sk

Comparison of the α-acceleration mechanism in helical MHD turbulence with the second-order Fermi mechanism is subject of the present contribution. The energetic spectra of accelerated energetic particles is investigated in following contribution [8].

II. THE KINETIC EQUATION

Owing to very high conductivity of the SW, the magnetic field is frozen-in into SW plasma which moves with the velocity $\boldsymbol{u}$. Thus the electric field $\boldsymbol{E}$ acting on the charged particle is

$$\boldsymbol{E} = -\frac{1}{c}[\boldsymbol{u}, \boldsymbol{H}] \,. \tag{1}$$

Let us separate the magnetic field $\boldsymbol{H}$ and plasma velocity $\boldsymbol{u}$ to homogeneous (regular) components $\boldsymbol{H}_0$, $\boldsymbol{u}_0$ and fluctuating (random) components $\boldsymbol{H}_1$, $\boldsymbol{u}_1$,

$$\boldsymbol{H} = \boldsymbol{H}_0 + \boldsymbol{H}_1 \,, \qquad \boldsymbol{u} = \boldsymbol{u}_0 + \boldsymbol{u}_1 \,. \tag{2}$$

Therefore, the Lorentz force possess beside the regular component also random component

$$\boldsymbol{F}_1 = e\boldsymbol{E}_1 + \frac{e}{c}[\boldsymbol{v}, \boldsymbol{H}_1] \,, \tag{3}$$

where the stochastic electric field has the form

$$\boldsymbol{E}_1 = -\frac{1}{c}\left([\boldsymbol{u}_0, \boldsymbol{H}_1] + [\boldsymbol{u}_1, \boldsymbol{H}_0] + [\boldsymbol{u}_1, \boldsymbol{H}_1] - \langle[\boldsymbol{u}_1, \boldsymbol{H}_1]\rangle\right) . \tag{4}$$

The angle brackets denote the averaging over the statistical ensemble of fields. Let us introduce the correlation tensor of random forces,

$$D_{\alpha\beta}(\boldsymbol{r}, \boldsymbol{p}, t; \boldsymbol{r}_1, \boldsymbol{p}_1, t_1) = \langle F_{1\alpha}(\boldsymbol{r}, \boldsymbol{p}, t) F_{1\beta}(\boldsymbol{r}_1, \boldsymbol{p}_1, t_1)\rangle \,. \tag{5}$$

Then averaged in the ensemble of the small scale fluctuation exact kinetic equation by usual averaging procedure [7], one finds in result the kinetic equation for the averaged (mean) CR distribution function $\mathcal{F} = \langle f \rangle$ [13], [7],

$$\frac{\partial \mathcal{F}}{\partial t} + \boldsymbol{v}\frac{\partial \mathcal{F}}{\partial \boldsymbol{r}} + \boldsymbol{F}_0\frac{\partial \mathcal{F}}{\partial \boldsymbol{p}} = \frac{\partial}{\partial p_\alpha}\overline{D}_{\alpha\beta}\frac{\partial \mathcal{F}}{\partial p_\beta} \,, \tag{6}$$

where

$$\overline{D}_{\alpha\beta} = \int_0^\infty \mathrm{d}\tau D_{\alpha\beta}(\boldsymbol{r}, \boldsymbol{p}, t; \boldsymbol{r} - \boldsymbol{v}\tau, \boldsymbol{p}, t - \tau) \,. \tag{7}$$

The mean electric field $\boldsymbol{E}_0$ is determined by correlator of the random medium velocity $\boldsymbol{u}_1$ and the stochastic MF $\boldsymbol{H}_1$. It is known that in an anisotropic turbulent medium the large-scale electric field appears which is directed along regular MF (so-called α-effect) [14], [27]. thus in this case the fields $\boldsymbol{u}_1$ and $\boldsymbol{H}_1$ are correlated:

$$\langle[\boldsymbol{u}_1, \boldsymbol{H}_1]\rangle = \alpha \boldsymbol{H}_0 \,. \tag{8}$$

Here the quantity α has dimensionality of velocity and is proportional to the helicity of turbulent medium. Consequently, owing to α-effect the following large-scale electric field is generated in a turbulent medium:

$$\boldsymbol{E}_{(\alpha)} = -\frac{\alpha}{c}\boldsymbol{H}_0 \,. \tag{9}$$

When the energetic particle scattering on magnetic irregularities is sufficiently intense so that the CR distribution function is near to isotropic, the diffusion approximation can be performed. Thus one can obtain the transport equation for CR density, $N = \int \mathrm{d}\Omega\, \mathcal{F}(\boldsymbol{r}, \boldsymbol{p}, t)$, where $\mathcal{F}$ is integrated over the angular variables of particle velocity. This transport equation has been obtained in previous paper [7].

III. THE MOMENTUM DIFFUSION COEFFICIENT

Here we consider only homogeneous case when the CR distribution function is independent on spatial coordinates. Performing the diffusion approximation of (6) one can obtain

$$\frac{\partial N}{\partial t} - \frac{1}{p^2}\frac{\partial}{\partial p} p^2 D_p \frac{\partial N}{\partial p} = Q \,. \tag{10}$$

Here we added the particle source Q and D_p presents the diffusion coefficient in the space of absolute value of particle momentum. This equation has a known form in stochastic acceleration theory [25], [26] and it is widely used in various astrophysical application. The momentum diffusion coefficient D_p can be written as a sum [7]

$$D_p = D_F + D_K \,, \tag{11}$$

where

$$D_F = \beta\,\frac{p^2\langle u_1^2\rangle}{3v\Lambda} \,, \quad D_K = \alpha^2 \frac{p^2\Lambda}{3vR_H^2} \,. \tag{12}$$

The first term D_F [26] describes the statistical Fermi acceleration [9] due to energetic particle scattering on moving magnetic irregularities. Here β depends on the relation between energy of regular H_0^2 and random $\langle H_1^2\rangle$ magnetic fields [7]. In most cases the regular and random MF have the same order of magnitude. We can believe that $\beta \approx 1$. The second term D_K defines particle acceleration by the large-scale electric field $\boldsymbol{E}_{(\alpha)}$ arising in the turbulent medium due to α-effect [12], [7]. Here $R_H = pc/eH_0$ is the proton Larmour radius. The relative efficiency of α-acceleration (in comparison to the second order Fermi mechanism) is given by the ratio

$$\frac{D_K}{D_F} = \frac{\alpha^2}{\langle u_1^2\rangle}\left(\frac{\Lambda}{R_H}\right)^2 \,. \tag{13}$$

The value of $\alpha^2/\langle u_1^2\rangle$ is a measure of gyrotropy in the turbulence and it is usually much less than unity in cosmic plasma. However the particle mean free path Λ can significantly exceed the Larmour radius.

The magnetic helicity of solar wind plasma was repeatedly measured on Voyager 1, 2, and Helios 1, 2. The mean free path in interplanetary magnetic field (IMF) exceeds Larmour radius more than on two order of magnitude in the energy region from several MeV up to some hundreds of MeV. Thus the dimensionless quantity (13) can be much more than unity in the solar wind plasma.

The magnetic helicity of the solar corona is extensive investigated currently. It is shown that the main contribution to coronal magnetic helicity is caused by solar active regions and the most important helicity injection in corona is caused by emergence of photospheric magnetic flux [3], [11], [24]. The theoretical investigations of α in solar convective zone allow to obtain the estimate value of $\alpha^2/\langle u_1^2\rangle \simeq 10^{-2} - 10^{-3}$ [21]. The ratio $\Lambda/R_H \geq 100$ for the proton energy range from MeV up to hundreds of MeV [6]. Therefore the ratio (13) specifying the relative efficiency of α-acceleration can be considerable (more than unity) in many astrophysical objects, e.g. solar corona, solar wind, supernova remnants, and so on.

In what follows we will examine only monoenergetic particle injection. The quantity of (13) under given injection energy has the form

$$\eta = \frac{\alpha^2}{\langle u_1^2\rangle}\left(\frac{\Lambda_0}{R_{0H}}\right)^2 , \tag{14}$$

where Λ_0, R_{0H} corresponds to the momentum p_0 of the particle injection. Note that dimensionless parameter η can be more (or even much more) than unity.

Let us suppose that particle mean free path has power law dependence on momentum,

$$\Lambda = \Lambda_0 \left(\frac{\zeta}{\zeta_0}\right)^{\lambda} , \qquad \zeta = \frac{p}{mc} \,, \tag{15}$$

where ζ defines the dimensionless particle momentum and m is the proton rest mass. Here ζ_0 corresponds to the injection momentum p_0. Then the momentum diffusion coefficient (12) related the Fermi stochastic acceleration can be written as

$$D_F = D_{0F}\zeta^{1-\lambda}\sqrt{1+\zeta^2} \,, \; D_{0F} = \frac{m^2c\langle u_1^2\rangle\zeta_0^{\lambda}}{3\Lambda_0} \,. \tag{16}$$

In the case of α-acceleration the coefficient D_K is

$$D_K = D_{0K}\zeta^{\lambda-1}\sqrt{1+\zeta^2} \,, \; D_{0K} = \frac{m^2c\,\alpha^2\Lambda_0\zeta_0^{2-\lambda}}{3R_{0H}} \,. \tag{17}$$

In the *nonrelativistic* range ($\zeta \ll 1$)

$$D_F \propto \zeta^{1-\lambda} \,, \quad D_K \propto \zeta^{\lambda-1} \,,$$

and for *ultrarelativistic* particles ($p \gg mc$) it follows from (16) and (17) that

$$D_F \propto \zeta^{2-\lambda} \,, \quad D_K \propto \zeta^{\lambda} \,.$$

One can see that for $\Lambda \propto \zeta^{\lambda}$, the momentum diffusion coefficient prove to be the power law function of momentum in the both energy ranges. It is worth to note that stochastic acceleration of transrelativistic CR has been investigated in [23].

The diffusion coefficients (12) depend essentially on the particle mean free path Λ (15). So, if Λ increases with proton kinetic energy ($0 < \lambda < 1$) then diffusion coefficient D_F for the second order Fermi mechanism is also rising function of momentum. The quantity (17) in this case decreases with kinetic energy in nonrelativistic range, and it is increasing function of ζ for ultrarelativistic particles.

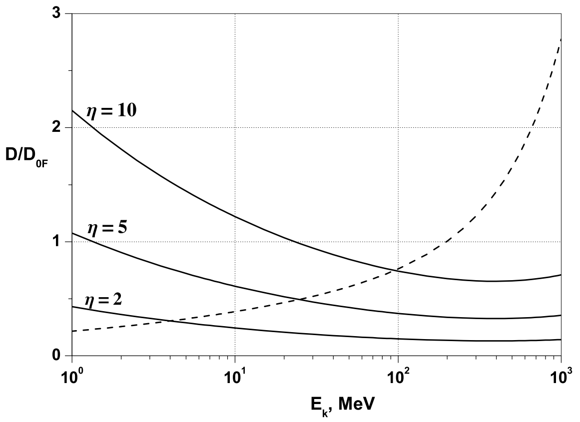

Fig. 1. *The momentum diffusion coefficient D_p dependence on kinetic energy for $\lambda = 0.5$.*

The dependence of D_p on the particle kinetic energy ε_k ($\varepsilon_k = \varepsilon - mc^2$, where ε is total energy) in D_{0F} units is shown in Fig. 1 for $\Lambda \propto \sqrt{p}$, ($\lambda = 0.5$). The Fermi acceleration coefficient D_F is shown by the dash curve, see (16). The solid curves are depicted for values of D_K/D_{F0} describing α-acceleration, see (17). Corresponding values of η, (14), are written near the curves. In our case of $\lambda = 0.5$ the α-acceleration is more efficient for low energetic particles while the stochastic Fermi acceleration dominate in high energy range. For example, if $\eta = 10$ both mechanisms have nearly the same efficiency at proton kinetic energy $\varepsilon_k = 100$ MeV.

When the proton mean free path has the power law dependence on momentum (15), the relative efficiency of α-acceleration (13) became the form

$$\frac{D_K}{D_F} = \eta \left(\frac{\zeta}{\zeta_0}\right)^{2(\lambda-1)}, \tag{18}$$

One can see that at $\lambda = 1$, ($\Lambda \propto p$), this ratio is independent on particle energy; when $\lambda > 1$, (or $\lambda < 1$), the ratio D_K/D_F grows (or decreases) with particle momentum.

The dependence of the relative efficiency (18) is demonstrated in Fig. 2 for the exponent of the mean free path equal to $\lambda = 0.5$ and several values of η (denoted near curves). The relative efficiency of acceleration by α-effect decreases when particle kinetic energy grows. Thus Fermi acceleration appears to be more effective for high energy CR. The magnitude of proton kinetic energy provided the equal efficiency of both acceleration mechanisms depends on the value of parameter η. For example, if $\eta = 10$, this typical energy equals 100 MeV, whereas for $\eta = 2$, the equality $D_F = D_K$ becomes at $\varepsilon_k = 40$ MeV.

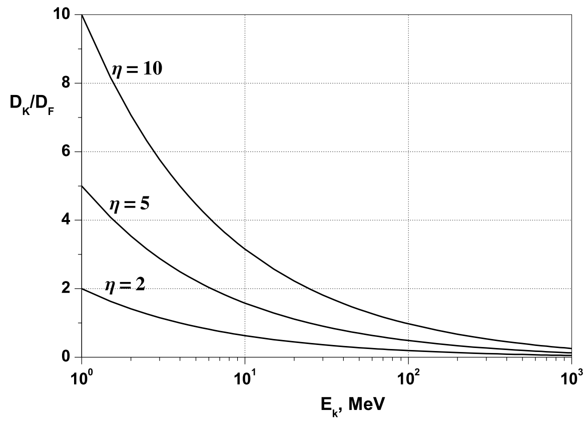

Fig. 2. *The relative efficiency (18) of α-acceleration for $\lambda = 0.5$.*

IV. THE INSTANTANEOUS INJECTION

Let us consider the CR energetic spectrum evolution in the case of instantaneous monoenergetic particle injection. Then the particle source in Eq.(10) reads

$$Q = \frac{\delta(p - p_0)\delta(t)}{p^2}. \tag{19}$$

The momentum diffusion coefficient (16),(17) in equation (10) prove to be a power law function of momentum for nonrelativistic and ultrarelativistic particles provided the mean free path is defined by Eq. (15), i.e.

$$D_p = D_0 \zeta^\gamma \tag{20}$$

holds. Note that in the case of Fermi mechanism $\gamma = 1 - \lambda$ (or $\gamma = 2 - \lambda$) for nonrelativistic (or ultrarelativistic) particles. On the other hand, in the case of α-acceleration $\gamma = \lambda - 1$ (or $\gamma = \lambda$) for nonrelativistic (or ultrarelativistic) particles. Taking into account expressions (19),(20) the Eq. (10) reads

$$\frac{\partial N}{\partial \tau} - \frac{1}{\zeta^2}\frac{\partial}{\partial \zeta}\zeta^{2+\gamma}\frac{\partial N}{\partial \zeta} = \frac{\delta(\zeta - \zeta_0)\delta(\tau)}{(mc)^3\zeta^2}, \tag{21}$$

where τ is dimensionless time:

$$\tau = \frac{t}{t_0}, \qquad t_0 = \frac{(mc)^2}{D_0}. \tag{22}$$

Solution of this equation by the method of Laplace transform yields the expression for evolution of temporal - momentum distribution of particles past the instantaneous monoenergetic injection,

$$N(\zeta,\tau) = \frac{\exp\left[-\frac{1+\gamma}{2}\ln(\zeta\zeta_0)\right]}{(2-\gamma)(mc)^3\tau}\exp\left(-\frac{\zeta^{2-\gamma}+\zeta_0^{2-\gamma}}{(2-\gamma)^2\tau}\right) I_\nu\left(\frac{2\exp\left[\tilde{\gamma}\ln(\zeta\zeta_0)\right]}{(2-\gamma)^2\tau}\right), \tag{23}$$

where $\tilde{\gamma} = (2-\gamma)/2$. The evolution of particle distribution in the momentum space is illustrated on Fig. 3 for instantaneous injection (23) and $\gamma = 0.5$ in the diffusion coefficient (20) for the Fermi acceleration mechanism (using (22),(16)). This value of γ corresponds to the mean free path proportional to

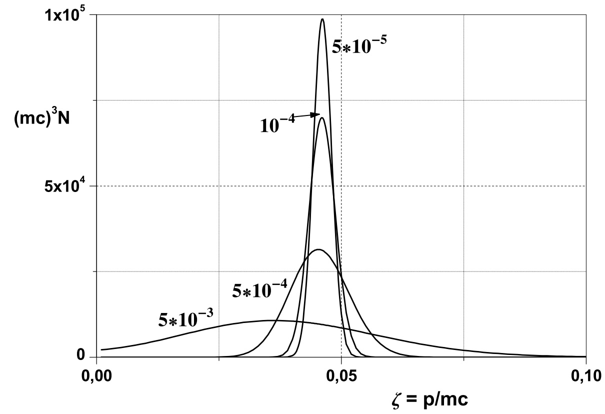

Fig. 3. *Evolution of particle density for $t_0 = 6$ sec, $\gamma = 0.5$ upon the Fermi acceleration.*

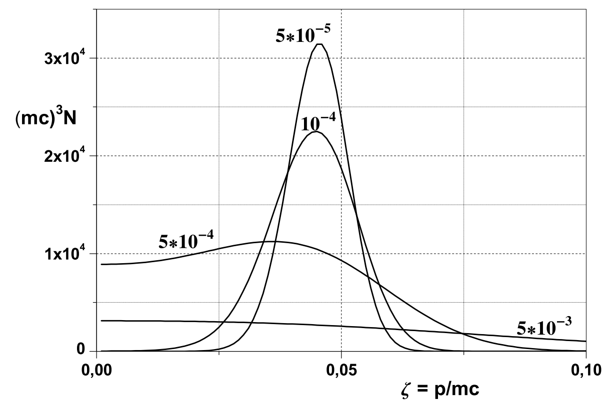

Fig. 4. *Evolution of particle density for $t_0 = 6$ sec, $\gamma = 0.5$ upon the α-acceleration.*

$\sqrt{p}$. The injection momentum p_0 corresponds to proton kinetic energy $\varepsilon_{k0} = 1$ MeV. Number near the curves are equal to the time interval since particle ejection (in seconds). Time $t_0 = 6$ seconds corresponds, for example, to the energy $\varepsilon = 1$ MeV, the magnetic field in acceleration region $H_0 = 100$ gauss [19], [6], the mean free path of 1 MeV proton $\Lambda_0 = 100 R_0$ [6], [17] and, the random medium velocity u_1 is accepted to equal Alfvenic speed $u_A = 10^8$ cm/s [6], [18], [16], [19]. The initial δ-like momentum distribution (19) became more and more smooth shape and the maximum of distribution shifts to the low energy region.

In the next Fig. 4 there is also particle dependence on dimensionless momentum ζ. As in Fig. 3, the mean free path has exponent equal $\lambda = 0.5$, but the momentum diffusion coefficient D_p has the form (17) fitting the α-acceleration process. In this case $\gamma = \lambda - 1$ in (20), so, $\gamma = -0.5$ for nonrelativistic particles. The acceleration time t_0 according (22),(17) was calculated for $\eta = 10$, and other parameters are the same as in Fig. 3. Here the evolution in momentum distribution occurs more rapidly due to α-acceleration than in the Fermi mechanism. This effect is caused by greater magnitude of acceleration time for the second order Fermi acceleration in the considered energy region in the case of chosen parameter $\eta = 10$.

V. CONCLUSION

The statistical acceleration of cosmic rays by anisotropic plasma turbulence is investigated. It is shown that the relative efficiency of α-acceleration is determined by the medium helicity and by the mean free path dependence on particle energy. The relative efficiency of acceleration by α-effect decreases when particle kinetic energy grows. It has been shown that evolution in momentum distribution occurs more rapidly due to α-acceleration than in the Fermi mechanism. The α-acceleration appears to be more effective for low energy CR and can by considered as an initial acceleration mechanism in helical turbulent environment. Energy spectra of accelerated particles and the energy spectra evolution is subject of the next contribution [8].

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