On the wavy heliospheric current sheet in the 2D transport equation for the galactic cosmic rays

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Abstract— **Extending our previous efforts we discuss the 2D transport equation (in space coordinates: radial distance - co-latitude) for the galactic cosmic ray intensity averaged over the longitude and over the period of the solar rotation. Some questions are considered concerning the solution of the 2D equation simulating some effects of the wavy current sheet on the GCR intensity and the ways to construct the 2D equation incorporating these effects to a greater extent. The expressions for the averaged over the longitude magnetic drift velocity are derived. The behavior of these velocities is compared with those of other authors and the advantages and shortcomings of the different approaches are discussed.**

I. INTRODUCTION

The importance and actuality of studying the effects of the global heliospheric wavy current sheet (WCS) on the GCR intensity are evident, especially in the present epoch, the solar activity minimum with the heliospheric magnetic field (HMF) polarity $A < 0$, when these effects should be most pronounced and both the GCR nucleonic intensity and the tilt of the current sheet vary rather unusually (see [1]).

Besides, in the epochs of low solar activity the longitudinal variation of the GCR intensity in the inner heliosphere is very small, i. e., its variation is mainly two-dimensional (2D; r, θ). However it is believed that the proper modeling of the WCS effects on the GCR intensity is possible only by solving the 3D (r, θ and the longitude φ) equation, [2]. Nevertheless, the major part of the GCR modeling in the inner heliosphere is fulfilled using the 2D models, simulating the WCS effects in different ways, [3]-[6].

Here we further develop our approach, [7]-[9], to get and solve the 2D transport equation for the GCR intensity averaged over the longitude. First, in Section II, starting from the full 3D equation we derive the equivalent set of two equations: 1) the 2D equation for the intensity averaged over the longitude (or over the period of solar rotation) and 2) the 3D equation for the variation of the intensity, the difference between the actual intensity and the averaged one. The advantages of this procedure are discussed. Then some questions are considered concerning the solution of the simplified 2D equation simulating the effects of the wavy current sheet on the GCR intensity and the ways to construct the 2D equation incorporating these effects to a greater extent. In Section III we derive the expressions for the averaged over the longitude magnetic drift velocities of the particles (both regular and current sheet ones) for general WCS surface and for the simple tilted current sheet (TCS) case. The results for the TCS case are compared with those of other authors and the advantages and shortcomings of different approaches are discussed.

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II. MAIN EQUATIONS

Note that below, discussing different equations, we always call the function to be found "the intensity", $J(\mathbf{r}, p, t)$, although actually the equations are for the omnidirectional distribution function, $U(\mathbf{r}, p, t) = J(\mathbf{r}, p, t)/p^2$. The distribution of the cosmic ray intensity in the heliosphere is usually described by the well-known equation

$$\frac{\partial U}{\partial t} + \nabla\left[\left(\mathbf{V}_{sw} + \mathbf{V}_d\right)U\right] - \nabla(K\nabla U) - \frac{\nabla\mathbf{V}_{sw}}{3}p\frac{\partial U}{\partial p} = 0, \tag{1}$$

where K, $\mathbf{V}_{sw}$, $\mathbf{V}_d$ are the diffusion tensor, solar wind and particle drift velocities, respectively.

First we derive the 2D equation for the intensity averaged over the longitude, $\overline{U} = U - \delta U$, which is just equal to $\sim$ 27-day mean intensity for steady $\overline{U}$. The coefficients of the equation can be decomposed in a similar way ($\mathbf{V}_{sw} = \overline{\mathbf{V}}_{sw} + \delta\mathbf{V}_{sw}, \mathbf{V}_d = \overline{\mathbf{V}}_d + \delta\mathbf{V}_d, K = \overline{K} + \delta K, \nabla\mathbf{V}_{sw} = \overline{\nabla\mathbf{V}}_{sw} + \delta(\nabla\mathbf{V}_{sw})$). However, instead of formulating the equations for $\overline{U}$ and δU using the averaged coefficients and their variations, let us here for short rewrite the eq. (1) in abbreviated form as

$$OU = 0, \tag{2}$$

where $O \equiv \frac{\partial\cdot}{\partial t} + \nabla\left[(\mathbf{V}_{sw} + \mathbf{V}_d)\cdot\right] - \nabla(K\nabla\cdot) - \frac{\nabla\mathbf{V}_{sw}}{3}p\frac{\partial\cdot}{\partial p}$ is the operator acting on U and characterized by the set of the coefficients K, $\mathbf{V}_{sw}$, $\mathbf{V}_d$, so that the sets of the averaged coefficients and their variations can be denoted as $\overline{O} \equiv \frac{\partial\cdot}{\partial t} + \nabla\left[(\overline{\mathbf{V}}_{sw} + \overline{\mathbf{V}}_d)\cdot\right] - \nabla(\overline{K}\nabla\cdot) + ...$ and $\delta O \equiv \nabla\left[(\delta\mathbf{V}_{sw} + \delta\mathbf{V}_d)\cdot\right] - \nabla(\delta K\nabla\cdot) + ...$ Averaging (2) over the longitude one can get

$$\overline{O}\cdot\overline{U} = -\overline{\delta O\delta U} \tag{3}$$

Then by subtracting (3) from (2) the following equation for δU can be obtained

$$\overline{O}\delta U + \delta O\delta U = -\delta O\overline{U} + \overline{\delta O\delta U} \tag{4}$$

However, the GCR intensity changes with time not only due to the rotation of the Sun (as, e. g., HMF's polariry is fixed in the rotating frame), $\frac{\partial}{\partial t} = -\omega\frac{\partial}{\partial\varphi}$. In general the GCR intensity changes also due to the variations in the averaged over the longitude characteristics, modulating the intensity. So it is advantageous to derive the 2D equation for the GCR intensity averaged over the period of solar rotation T (that is, over the time during T). Besides, it is averaging over T, $\langle\rangle_T$, what we can easily get from the observations, not $\langle\rangle_\varphi$. Naturally, eqs. (3 - 4) don't change if we mean by the $\overline{U} = U - \delta U$ and $\overline{O} = O - \delta O$ the averaging over the period of solar rotation.

The set (3)–(4) with the boundary and initial conditions (not to be discussed here) is equivalent to (2). If one neglects the right-hand side in (3), this equation will describe the cosmic ray propagation in the longitudinally averaged heliosphere. This axisymmetric heliosphere may have strange features, e.g., non-divergence-free magnetic field, as was noted in [2]. It is just the source in the right-hand side of (3), describing the contribution to the average density from the asymmetrical parts of the modulation factors and intensity, which makes the 2D equation for $\overline{U}$ the exact one.

So if one knows δU and puts it in RHS of (3), this equation can be used not only for simulating the main effects of the WCS in the GCR intensity, but to model these effects in full measure. Of course, to find δU one should solve the 3D equation (4).

Let us discuss the analogues of the eqs. (3)-(4) for the simplest case when the only φ-dependent characteristic is the polarity of the regular magnetic field. Here we shall use the equations in full notation:

$$\frac{\partial \overline{U}}{\partial t} + \nabla[(\mathbf{V}_{sw} + \overline{\mathbf{V}}_d)\,\overline{U}] - \nabla(K\nabla\overline{U}) - \frac{\nabla \mathbf{V_{sw}}}{3} p \frac{\partial \overline{U}}{\partial p} = -\left\{\nabla \overline{\delta \mathbf{V}_d \delta U}\right\} \quad (5)$$

$$\frac{\partial \delta U}{\partial t} + \left\{\propto \delta U, \frac{\partial \delta U}{\partial x_i}, \dots\right\} = -\left\{\nabla\left[\delta \mathbf{V}_d\,\overline{U}\right]\right\} \quad (6)$$

In the figure brackets in the left-hand side of (6) we collected all terms containing δU and its space and momentum derivatives.

In [7] - [9] we considered different approaches to estimate δU without solving the full (6). We have not achieved what we wanted although some useful features of the particle's behavior near the WCS have been revealed. We still anticipate that the above aim can be attained if one neglects the terms in the figure brackets in the LHS of (6) relative to its RHS.

However the more direct way to estimate the RHS terms in eq. (5) for $\overline{U}$ is to solve the initial 3D equation (1) for U, then construct from it $\overline{U}$ and δU and compare the terms in the RHS of (5) with those in the LHS of this equation. We are working on it.

III. AVERAGING MAGNETIC DRIFTS

The most simple (and widely used) 3D heliospheric model is that where the solar wind is radial with constant velocity and the only φ-dependent characteristic is the polarity of the regular magnetic field. Then the only term in the LHS of eq.(3) which needs averaging is that with $\mathbf{V}_d$ and the magnetic field can be represented as $\mathbf{B} = F \cdot \mathbf{B}_m$, where $\mathbf{B}_m$ is the unipolar (or "monopolar") magnetic field equal to $\mathbf{B}$ in the positive sectors and having the reversed polarity in the negative ones. F is a scalar function equal to $+1$ in the positive and -1 in negative sectors, changing from one to another on the CS surface $F(r, \vartheta, \varphi, t) = 0$.

The particle drift velocity is $\mathbf{V}_d = pv/3q \cdot \left[\nabla \times (\mathbf{B}/B^2)\right]$, [10], where v and q are the particle speed and charge, respectively. In our case one can decompose the drift velocity into the regular $\mathbf{V}_d^{reg}$ and current sheet $\mathbf{V}_d^{cs}$ ones,

$$\mathbf{V}_d^{reg} = \frac{pv}{3q} \cdot F \cdot \left[\nabla \times \frac{\mathbf{B}_m}{B^2}\right] \quad (7)$$

$$\mathbf{V}_d^{cs} = \frac{pv}{3q} \cdot \left[\nabla F \times \left(\frac{\mathbf{B}_m}{B^2}\right)\right] \quad (8)$$

In general $F = A\,[2H(x) - 1]$, with A and $H(x)$ being the polarity of the HMF in the high-latitude N-hemisphere and the step function of the distance x from the sheet, respectively. The value of x is positive in the N-"magnetic hemisphere". In this case $\nabla F = 2A\delta(x)\vec{\mathbf{n}}^{\mathbf{cs}}$,where $\vec{\mathbf{n}}^{\mathbf{cs}}$ is the unit vector normal to CS surface and directed to the N-hemisphere. Then the averaged over the longitude drift velocities are as follows

$$\overline{\mathbf{V}}_d{}^{reg} = \frac{pv}{3q} \cdot \overline{F} \cdot \left[\nabla \times \frac{\mathbf{B}_m}{B^2}\right] \quad (9)$$

$$\overline{\mathbf{V}}_d{}^{cs} = \frac{A}{\pi} \cdot \frac{pv}{3q} \cdot \frac{1}{B \cdot r \sin\vartheta} \cdot \sum_{i=1}^{i=2k} \left.\frac{\vec{\mathbf{n}}^{\mathbf{cs}} \times \vec{\mathbf{n}}_{\mathbf{b}}}{|\vec{\mathbf{n}}^{\mathbf{cs}} \cdot \vec{\mathbf{e}}_\varphi|}\right|_{\varphi_i}, \quad (10)$$

where $\vec{\mathbf{n}}_{\mathbf{b}}$ is the unit vector along the "monopole" magnetic field line and the summation is performed for all points ($\varphi_i, i = 1, \cdots, 2k$) of interception of the circle $r, \vartheta = const$ with the WCS surface. These expressions are valid for any WCS surface.

In case of generalized tilted current sheet (TCS) describing a $2k$-sector structure with the CS surface specified as in [7]

$$\vartheta^{cs} = \frac{\pi}{2} - \arctan\left(\tan\alpha\ \sin k(\varphi + \frac{\omega(r - r_{ss})}{V_{sw}})\right), \quad (11)$$

where ω and r_{ss} are the angular velocity of the Sun and the source surface of the HMF, respectively. Then the scaling factor for the regular drift velocity and the averaged over the longitude current sheet drift velocity look as follows

$$\overline{F} = \frac{2A}{\pi} \arcsin(\cot\alpha \cot\vartheta) \quad (12)$$

$$\overline{\mathbf{V}}_d^{cs} = \frac{2A}{\pi} \frac{pv}{3q} \frac{\sin\chi}{B \cdot r \sin^2\vartheta} \frac{\cot\alpha}{\sqrt{1 - \cot^2\alpha \cdot \cot^2\vartheta}}, \quad (13)$$

if ($\frac{\pi}{2} - \alpha < \vartheta < \frac{\pi}{2} + \alpha$), and $\overline{F} = \pm A, \overline{\mathbf{V}}_d{}^{cs} = 0$ outside this HMF sector-structure zone. As in [7] we call attention that neither $\overline{F}$, nor $\overline{\mathbf{V}}_d{}^{cs}$ depend on k. Note that although the expression (13) for $\overline{\mathbf{V}}_d{}^{cs}$ diverges at $\vartheta_0 = \frac{\pi}{2} - \alpha$, it makes it very slowly ($\overline{\mathbf{V}}_d{}^{cs} \propto (\vartheta - \vartheta_0)^{-\frac{1}{2}}$), so there is no jump in the latitudinal gradient of the intensity at this colatitude.

In Fig. 1 the colatitude, radial and energy profiles of the scaling factor of the regular magnetic drift, derived in this work (KK-model), are compared with those obtained in [3], [4] (PMB-model) and [5] (HB-model). Similarly, in Fig. 2 the colatitude, radial and energy profiles of the current sheet drift velocity derived in this work, are compared with those obtained in the PMB- and HB-models.

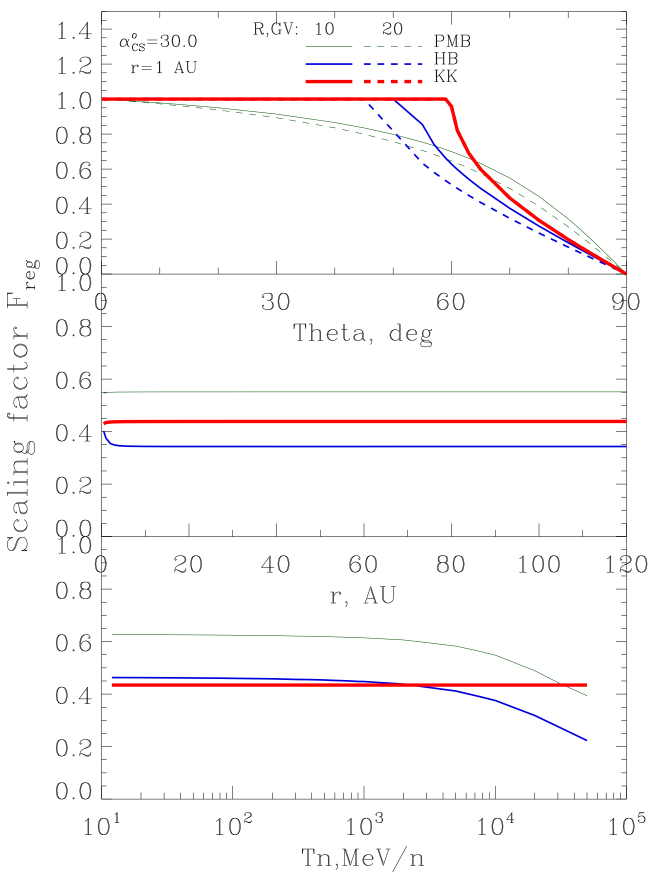

Fig. 1. The colatitude (upper panel), radial (middle panel) and energy (lower panel) profiles of the due to the WCS effects scaling factor of the regular magnetic drift velocities for three models. The colatitude profiles are shown for different rigidities of the charged particles.

One can see that
1) For our model the WCS effects in $\overline{F}$ and $\overline{\mathbf{V}}_d{}^{cs}$ exist only in the sector-structure zone, while those in PMB- and HB-models extend to higher latitudes, the higher latitude for the greater rigidity of the particles.
2) The latitude dependence of the current sheet drift velocity is different for three models considered: for our model $\overline{\mathbf{V}}_d{}^{cs}$ is greatest at the maximum extend of WCS, $\vartheta_0 = \frac{\pi}{2} - \alpha$; it is greatest at the equator for PM-model; and it does not depend on latitude for HB-model. Note that the results of the numerical model [6] (AUMK-model) are similar to ours in this

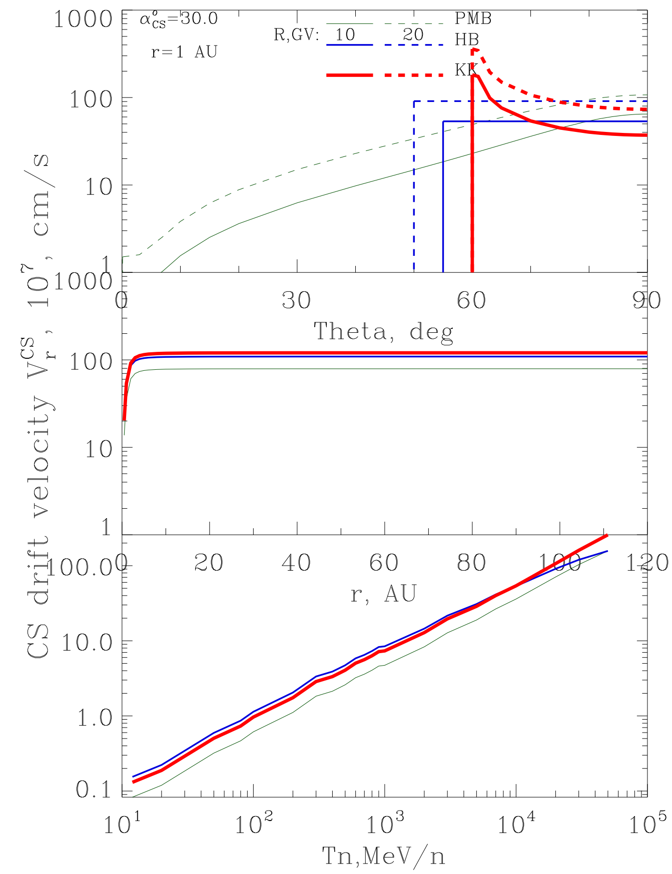

Fig. 2. The colatitude (upper panel), radial (middle panel) and energy (lower panel) profiles of the current sheet magnetic drift radial velocity for three models. The colatitude profiles are shown for different rigidities of the charged particles.

respect, at least partially.

We are sure that, because of the finite giroradius of the particles, there is a physical sense in the widening with rigidity of the latitude range where the particles feel the WCR, so our model should be modified in this respect. On the other hand the flat current sheet velocity field supposedly used in HB-model is valid only for homogeneous HMF (or particles with low rigidity) and formally it could not be used for high rigidity particles. The AUMK-model uses the complicated procedure to cope with this difficulty. Besides, in order to ensure that the resultant drift velocity is divergence-free, the current sheet drift velocity in HB-model was averaged over the latitude within the effective WCS latitude range. However, it could be misleading, if one is interested in the details of the GCR distribution in this latitude range.

Our opinion is that the main cause of the shortcomings of all models for the WCS drift velocity is the use of the infinitely thin current sheet. So we believe that the proper way to construct the consistent drift velocity field is to use more realistic model of the heliospheric current sheet. We are working on it.

IV. CONCLUSIONS

1) The proposed earlier procedure is recalled that allows one to get the precise 2D transport equation for the GCR intensity averaged over the longitude and, in principle, for the case of small variation of the intensity with the longitude, to estimate longitude dependent wave in the intensity without solving 3D boundary problem. Probably, to construct this precise 2D transport equation, the detailed 3D calculations are necessary.
2) The averaged over the longitude magnetic drift velocities are derived both for the general surface of the wavy current sheet and for the widely used tilted current sheet model. The comparison of the scaling factor of the regular magnetic drift velocities and the current sheet drift velocity with those of other authors shows the shortcomings of all models due, in our opinion, to the use of the model of the heliospheric magnetic field with the heliospheric current sheet of infinite thinness.

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