On the Calculations of Cosmic Ray Transmission Function

P.Bobik, K. Kudela and R. Bučík

Abstract—**Trajectories calculations of low energy cosmic rays in the models of geomagnetic field are widely used for estimation of the particle access either to ground stations or to satellites positions. There were many discussions about precision of these calculations especially in the penumbra regions. Transmission function is based on these calculations. We check hypothesis of convergence of transmission function from some level of calculation precision. We test this hypothesis in IGRF and Tsyganenko 96 models of geomagnetic field mainly for vertical direction. Influence of parameters of the calculation is tested..**

1. Introduction

Trajectory calculations of particles in the geomagnetic field is usual method to evaluate space distribution and origin of energetic particles inside the magnetosphere. Method has been already used for several decades [1]. Many discussions about precision of these calculations in so called penumbra region of magnetosphere was made [2]. All are based on the simple fact that small change in initial condition of calculation in penumbra region, can cause a allowed trajectory became forbidden or forbidden trajectory to became allowed. It can change for a example a classification of origin of a detected particle in the experiments, when they are evaluated by trajectory calculations analysis. In this article we show how this “chaotic” behavior can change or affect evaluated fluxes in penumbra regions for different levels of calculation precision.

2. Method

Usual approach to the trajectory calculations of cosmic rays in the geomagnetic field is based on the reversal of both the sign of the electric charge and the velocity of the particle in Lorentz equation. After this motion equation stay unchanged and we can obtain a same trajectory using in calculation antiparticle moving in opposite direction.

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$$m\frac{d\vec{v}}{dt} = Zq(\vec{v}\times\vec{B}) \qquad (1)$$

Where m is the relativistic mass and $\boldsymbol{v}$ is velocity of particle with charge Z. $\boldsymbol{B}$ is total magnetic field. We use a model of geomagnetic field which consist of magnetic field from internal (IGRF, see the http://modelweb.gsfc.nasa.gov/models/igrf.html) and external sources [3].

Initial conditions for every position in the magnetosphere are rigidity $\Re$ of particle calculated from its momentum and charge, incoming direction, and two parameters of the calculation. First is angle α between previous and next calculated particle vector and second is limit of all integration steps for one trajectory. First parameter describe how smoothly we follow particle trajectory in the magnetosphere and second one is introduced because some particles can be trapped by geomagnetic field.

Result of numerical integration of equation (1) is a particle trajectory in geomagnetic field. Following the Liouville theorem, this trajectory can be allowed or forbidden [4]. The trajectory is allowed when particle coming from magnetopause can reach selected point inside magnetosphere and forbidden when particle can not reach selected point. Calculation of trajectories for all possible directions of incoming particles and all energies give us information which can be used for evaluation a particle flux. In this paper we calculate flux using a energy spectra of Galactic Cosmic Rays outside the magnetosphere and transmission function which is constructed from set of calculated trajectories [5], [6], [7]. In reality we can for every incoming direction calculate trajectories for a set of energies not for any possible energy. We use energies separated by energy step ΔE (or rigidity step $\Delta\Re$). This introduce new parameter of calculation rigidity step $\Delta\Re$. Also directions must by selected with some steps in space angles. Outside of penumbra region all trajectories up to some energy border are forbidden, above this energy they are allowed. In the penumbra region there are low and up border energies which divide region of mixed allowed and forbidden trajectories from regions with all trajectories forbidden, or all trajectories allowed [8], [4]. Trajectories in the penumbra region are sensitive to initial condition, and this lead to question about possibility to evaluate precise transmission function and because that also about possibility evaluate precise flux. Same questions we have for cut off rigidities.

During trajectory calculations for one direction we calculate N trajectories with equidistant rigidities from range ($\Re$-d$\Re$/

2,ℜ+dℜ/2) with rigidity step $\Delta\Re = d\Re/N$. When we increase number of calculated trajectories in fixed dℜ (so decrease $\Delta\Re$ step between trajectories) and from obtained results evaluate cut-off rigidities or flux, we can check how big is influence of small change in initial ℜ to flux or cut-off rigidity. Because in the experiments we measure flux in energy bins we perform calculation not just for a single energy but for set of energies from energy bin. Because that there is a important result for full bin not for one specific energy. Then if results for bin are constant or if they are from some level of precision change less than is measurement error, we should not be so much afraid of trajectory calculation sensitivity to initial condition.

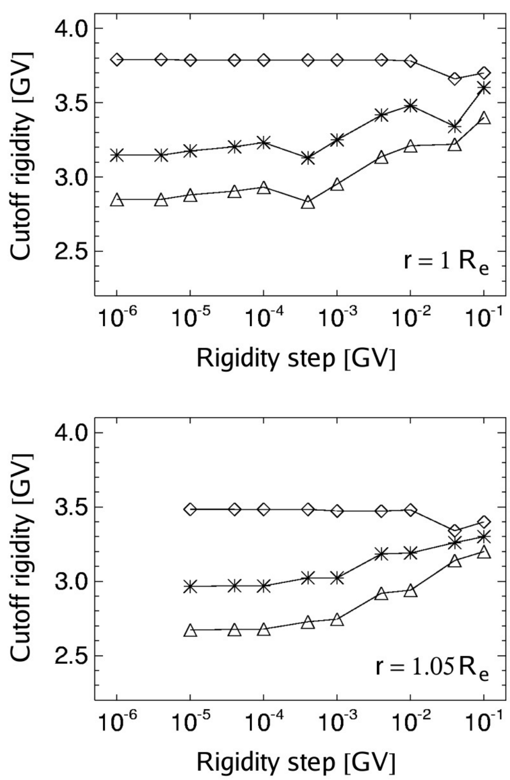

Figure 1. Cutoff rigidities for a middle latitude position (50.00 N, 50.00 E) at Earth surface (upper panel) and at a low orbit (bottom panel) calculated for a set of rigidity steps $\Delta\Re$.

3. Results and Intepretation

A. Middle latitude position

We calculate a sets of vertically incoming protons trajectories for two selected points at Earth surface and low orbit (altitude 400 km) and both in the penumbra region at middle latitude and in the equatorial region. Sets was calculated with different rigidity steps $\Delta\Re = 10^{-6}$, 4.10^{-5}, 10^{-5}, 4.10^{-4}, 10^{-4}, 4.10^{-3}, 10^{-3}, 4.10^{-2}, 10^{-2}, 4.10^{-1}, 10^{-1} GV. From these calculation we find a cutoff rigidities and evaluate fluxes.

Results for calculation in combined geomagnetic field from internal (IGRF) and external [3] fields are presented in figures 1. and 2.. In the figure 1. are cutoff rigidities (used cutoff terminology is adopted from [9]) for middle latitude position (50.00 N, 50.00 E) at Earth surface and at position at low orbit. Upper cut-off rigidity is stable from step $\Delta\Re < 10^{-2}$ GV. Effective cut-off rigidity dependency at $\Delta\Re$ show convergence to stable value from $\Delta\Re < 10^{-3}$ GV.

Figure 2. show fluxes for the same situation. The flux was evaluated using interplanetary spectrum $\Phi^{1AU}(\Re)$ based on AMS-01 measurements and IMP-8 measurements [10]. Flux at selected point is estimated accordingly to

$$\Phi(\Re) = \Phi^{1AU}(\Re)\,TF(\Re)$$
$$TF(\Re) = \frac{\sum_{i=1}^{N} P_i}{N} \qquad (2)$$
$$N = \frac{d\Re}{\Delta\Re}$$

Where $\Phi(\Re)$ is modulated spectrum inside the magnetosphere, and $TF(\Re)$ is transmission function evaluated for bins with width $d\Re$. In every bin (ℜ-dℜ/2,ℜ+dℜ/2) of transmission function we have N trajectories with probabilities P equal zero if trajectory is forbidden , or equal one if trajectory is allowed.

For $\Delta\Re < 10^{-3}$ GV the results showed at figure 2. are almost stable and variation in flux is smaller than 0.2 % what means variation in range less than ± 1 particle for $m^2 s^{-1} sr^{-1} GeV^{-1}$. At low orbit we have similar situation.

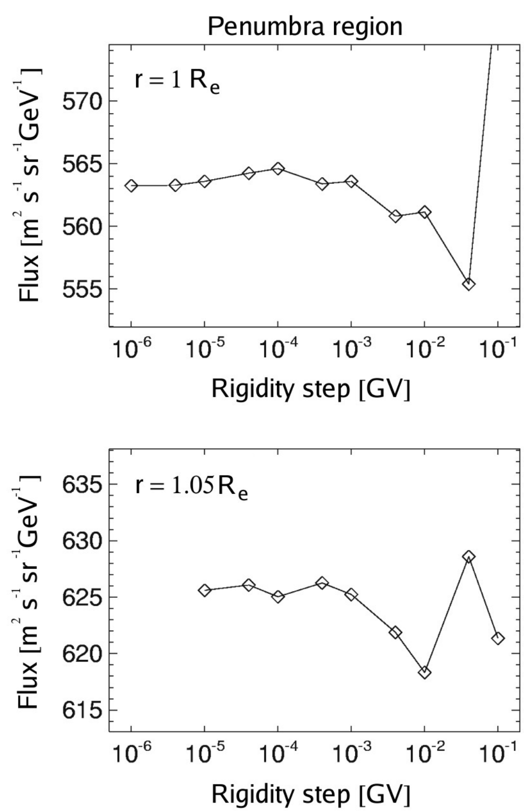

Figure 2. Fluxes for a middle latitude position (50.00 N, 50.00 E) at Earth surface (upper panel) and at a low orbit (bottom

panel) evaluated for a set of rigidity steps $\Delta\Re$.

B.Low latitude position

Low latitude positions have not a penumbra [8]. Outside a penumbra region we don't have a three rigidity just one cut-off rigidity (up of this border is any trajectory with higher energy allowed). For our calculation we choose position with very high cut-off rigidity (10.00 N, 95.00 E). Dependence of cut-off rigidity on $\Delta\Re$ is presented at figure 3. Cut-off rigidity is stable from step $\Delta\Re < 10^{-3}$ GV. For $\Delta\Re < 10^{-3}$ GV is variation of flux smaller than 0.01 % what means change a less than 0.001 particle for 1 $m^2s^{-1}sr^{-1}GeV^{-1}$ in flux variation. Flux dependence on $\Delta\Re$ at low latitude position is presented at figure 4.

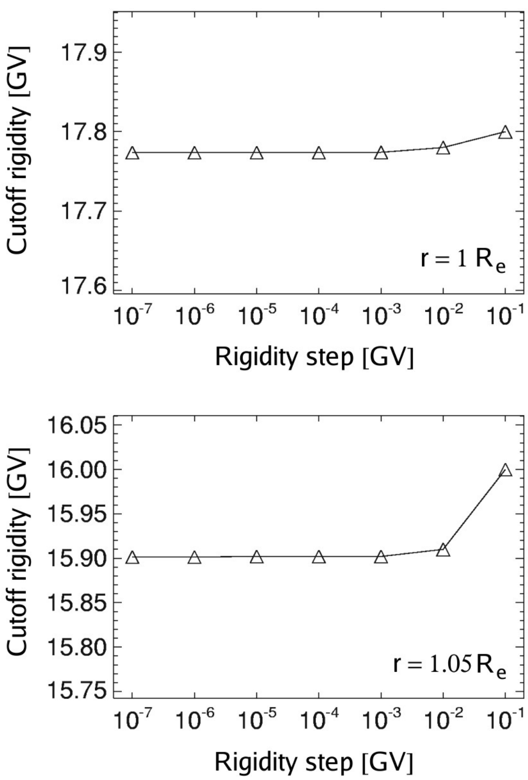

Figure 3. Cutoff rigidities for a low latitude position (10.00 N, 95.00 E) at Earth surface (upper panel) and at a low orbit (bottom panel) calculated for a set of rigidity steps $\Delta\Re$.

Showed results should be valuated in comparison with experiments results. For comparison we choose a AMS-01 experiment which was flown on the space shuttle Discovery during flight STS-91 in June 1998. The proton spectrum was measured in the kinetic energy range 0.1 to 200 GeV with acceptance of 0.15 m^2 sr on average. [11]. Generally for AMS-01 are errors in order of percent of measured flux [12]. Calculation errors in penumbra region, connected to flux change with increasing number of calculated trajectories are ten times lower. In low latitude position four orders lower.

To check influence of different models of geomagnetic field we perform same calculations only in geomagnetic field of internal sources (IGRF). For these calculations we obtain very similar results to results from geomagnetic combined field calculations in sense of convergence flux and rigidities to relatively stable value with decreasing a rigidity step $\Delta\Re$.

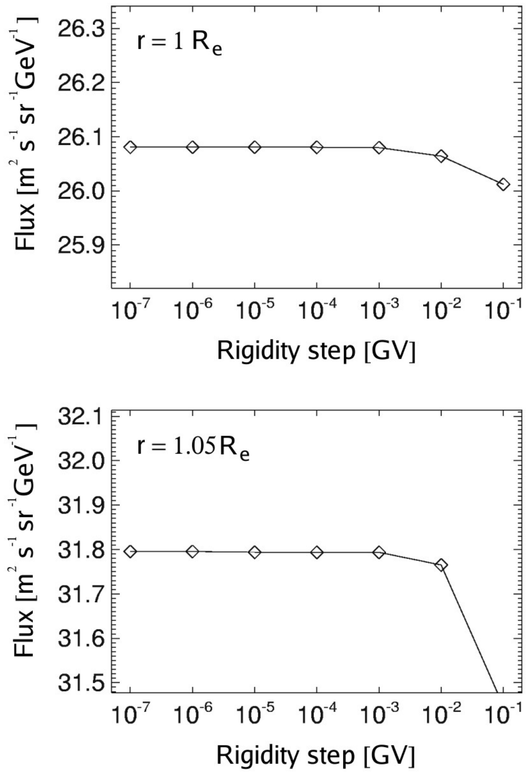

Figure 4. Fluxes for a low latitude position (10.00 N, 95.00 E) at Earth surface (upper panel) and at a low orbit (bottom panel) evaluated for a set of rigidity steps $\Delta\Re$.

C.Non vertically incoming particles

More realistic evaluation of flux can be done with calculation of all possible directions of incoming particles to selected point. However, if we want perform similar test like was presented here for all possible incoming direction, substituted by a net of incoming directions, our calculation needs in orders more demand of computing capacity than same calculations for vertically incoming particles. Such set of calculations is now behind of usual available computing capacity. Preliminary calculations for the same positions as was used for vertically incoming particles for $\Delta\Re = 10^{-3}$, 10^{-2}, 10^{-1} GV for 145 incoming directions for every selected point shows same kind precision convergence as for vertically directed trajectories.

D.Alternative approach

Alternative approach to equidistant rigidity steps is generate initial rigidities randomly with uniform distribution over rigidity. Random generation of energies tell us how small changes in initial energy can affect evaluated flux. We made test calculation for *N* randomly generated particles to energy range of every bin. Results for N = 10^1, 10^2,10^3,10^4,10^5 particles for one energy bin for middle latitude position (50.00 N, 50.00 E) at Earth surface are at figure 5. Because for our transmission function we use energy bins 0.1GV, N = 10^2 particles injected uniformly to one bin is equivalent to calculation with equidistant step $\Delta\Re = 10^{-3}$ GV. Presented results are very similar to results with equidistant step. They produce almost same fluxes and same type of convergence than calculations with equidistant steps. This is proof that from some level of calculation precision we are in flux estimation not sensitive to small changes of initial rigidity of particle.

E. Angle α influence to flux evaluation

We made a set of test calculation for a calculation parameter α at the middle latitude position (50.00 N, 50.00 E) in a range from $\alpha = 5.10^{-4}$ to 10^{-1} radians. From these calculation we evaluate a fluxes. Fluxes converge to stable value (changes in intensity less than 1% for $m^2s^{-1}sr^{-1}GeV^{-1}$) for $\alpha < 2.10^{-3}$ rad.

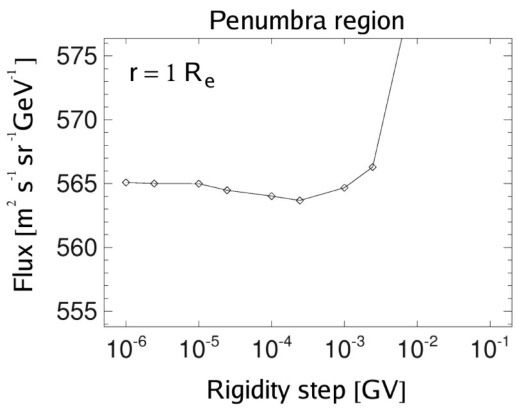

Figure 5. Fluxes for a middle latitude position (50.00 N, 50.00 E) at Earth surface (upper panel) and at a low orbit (bottom panel) evaluated for a sets of uniformly generated rigidities (in average equivalent to set of steps $\Delta\Re$).

6. Conclusion

Question of trajectory calculations precision in the penumbra region in connection to calculation sensitivity to small changes of initial condition can be answered in following way and depend on definition what we mean under precise calculation. We define precise calculation, to have connection to errors of actual measurements, as calculation where particle flux with increasing numerical precision of calculation change less than 1% in flux for $m^2s^{-1}sr^{-1}GeV^{-1}$. After we can conclude that is possible make precise calculation in the penumbra region. From rigidity step $\Delta\Re < 10^{-3}$ GV results in penumbra region change minimally and flux and cut-off rigidities converge to relatively stable value. Spectrum of allowed and forbidden trajectories became from mentioned level of calculation precision self-similar. Under self-similarity we mean that almost same percent of all trajectories in penumbra are allowed and forbidden when we make calculation with more and more trajectories. Penumbra structure self - similarity lead to convergence in flux from some level of calculation precision in rigidity step.

Acknowledgments

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