GALACTIC COSMIC RAY – CLOUDS EFFECT AND BIFURCATION MODEL OF THE EARTH GLOBAL CLIMATE

V.D. Rusov, A.V. Glushkov, V.N. Vaschenko, O.T. Myhalus, S.I. Kosenko, S.Ch. Mavrodiev, B. Vachev

Abstract— **The possible physical linkage between the cosmic rays – cloud and indirect aerosol effects is discussed using the analysis of the first indirect aerosol effect (Twomey effect) and its experimental representation as the dependence of mean cloud droplet effective radius versus aerosol index defining the column aerosol number. It is shown that the main kinetic equation of Earth climate energy-balance model is described by the bifurcation equation (relative to the surface temperature of the Earth) in the form of fold catastrophe with two controlling parameters defining the variations of insolation and Earth magnetic field (or cosmic rays intensity in the atmosphere) respectively. The results of comparative analysis on the time-dependent solution (time series of global palaeotempreture) of Earth climate energy-balance model taking into account nontrivial role of galactic cosmic rays and the known experimental data on the palaeotemperature from the EPICA Dome C and Vostok ice core are presented.**

1. Introduction

The fact that galactic cosmic rays (GCR) play one of key parts in the mechanisms responsible for the weather and climate variations observed at our planet is highly plausible [1]. Summarizing the outcomes of numerous studies (see, e.g. [1,2]) concerned with the influence of cosmic ray flux (CRF) on atmospheric processes, particularly on the formation of aerosols (condensation centres of water vapour), the following causal sequence of events can be appointed: brighter sun → modifications of solar activity and insolation → modulation of galactic CRF → changes of cloudiness and thunderstorm activity → change of albedo → variations of weather and climate.

V.D. Rusov is with the Odessa National Polytechnic University, Odessa, 65044 Ukraine (corresponding author to provide phone: 380-482-64172; fax: 380-482-64172; e-mail: siiis@te.net.ua).

A.V. Glushkov is with Odessa State Environmental University, Odessa, 65106 Ukraine (e-mail: glushkov@paco.net).

V.N. Vaschenko is with the Ukrainian National Antarctic Scientific Center, Kiev, 01601 Ukraine (e-mail: daniilko@mail.ru).

O.T. Myhalus and S.I. Kosenko are with the Odessa National Polytechnic University, Odessa, 65044 Ukraine (e-mail: skose@te.net.ua).

S.Ch. Mavrodiev and B. Vachev are with the Institute for Nuclear Research and Nuclear Energy, Sofia,1784 Bulgaria (e-mail: mavrodi@inrne.bas.bg).

This paper have for an object a development of energy-balance model of climatic response to orbital variations, which takes into account an influence of galactic cosmic rays on global climate.

2. On possible relation between cosmic-ray-cloud and indirect aerosol effects

It is known, that the indirect observation of Twomey effect (the first indirect aerosol effect) can be made by comparing cloud droplet size and aerosol concentration. In fact, a dependence of CDR on AI (Fig. 1) is measured in actual satellite observations by means of radiometers [3]. This is determined by the fact that "CDR is more sensitive to the aerosol index than to the optical thickness, which is to be expected, because the aerosol index is function of the CCN (cloud condensation nuclei) concentration" [3]. It can be easily shown that the observed dependence of CDR on AI over oceans and land is represented (with approximation sufficient for our purpose) as the empirical dependence

$$AI = \left[\frac{1}{(0.6r_{eff} - 4.385)r_{eff}} - \frac{\eta}{r_{eff}}\right]^{1.429}, \quad \eta = \begin{cases} 0, & over \quad ocean, \\ 0.63, & over \quad land, \end{cases} \qquad (1)$$

where $r_{eff} = \langle r^3 \rangle / \langle r^2 \rangle$ is the mean CDR, and r is the radius of the cloud droplets.

Further on we will use the fact the number of aerosol particles that may act as CCN, N_{CCN}, and the number of cloud droplets, N_d , are approximately related through [4]

$$N_d \approx (N_{CCN})^{\alpha}. \qquad (2)$$

Taking into account that, on the one hand, cloud process models and measurements indicate that α is on the order of 0.7 [3], and, on the other hand, $AI \sim N_{CCN}$ [3], using Eqs. (1) and (2) we introduce the following expression for the concentration of cloud droplets:

$$N_d \approx \frac{1}{(0.6r_{eff} - 4.385)r_{eff}} - \frac{\eta}{r_{eff}}. \qquad (3)$$

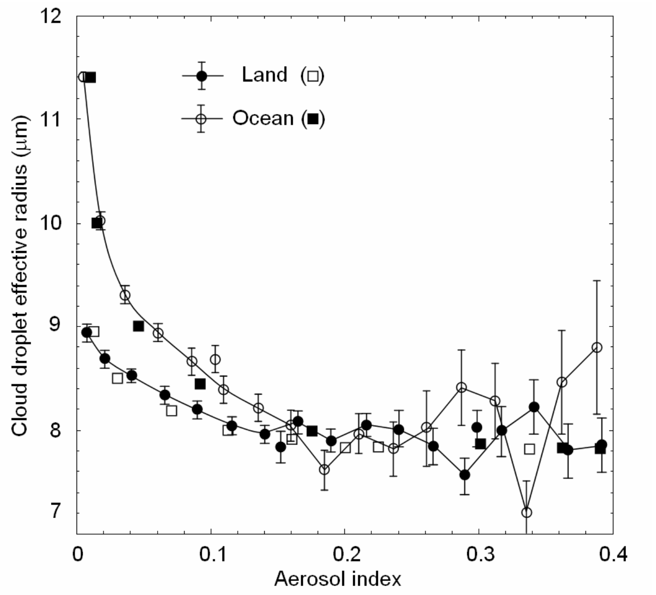

Fig. 1. Effect of aerosol on cloud droplet: mean cloud droplet effective radius (CDR) as a function of aerosol load [3]. The two curves show the mean CDR as a function of aerosol index (AI) for land (lower curve) and ocean (upper curve). The error bars represent the confidence level of the mean value, i.e., $\sigma/\sqrt{n-2}$, where n and σ are the number of CDR measurements within the bin and their standard deviation [3]. The values of empirical relation CDR=f (AI) in the form of (1) for land (■) and ocean (□) are presented.

Then the volume of liquid water in the atmosphere "over the ocean" or "over the land", V_W, equals to

$$V_W = p_i V_{atm} \cdot \frac{4}{3}\pi \langle r \rangle^3 N_d = p_i V_{atm} \cdot \frac{4\pi}{3}\frac{r_{eff}^2}{k_r^3}\left(\frac{1}{0.6 r_{eff} - 4.385} - \eta\right), \quad (4)$$

where $V_{atm} \cong const$ is the total volume of the atmosphere, p_i is the portion of the atmosphere volume "over the ocean" or "over the land", $\langle r \rangle \approx k_r \cdot r_{eff}$ is the mean radius of cloud droplets. Therefore, the averaged total volume of liquid water in the atmosphere can be defined as

$$\langle V_W \rangle = V_{atm} \cdot \frac{4\pi}{3}\frac{r_{eff}^2}{k_r^3}\left[\frac{1}{0.6 r_{eff} - 4.385} - \frac{S_{land}\Lambda}{S_{ocean} + S_{land}} 0.63\right], \quad (5)$$

$$\Lambda = \Lambda(r_{eff} - 7.7) - \Lambda(r_{eff} - 9.6),$$

where $\Lambda(x)$ is the integral of δ-function

$$\Lambda(x) = \int_{-\infty}^{x} \delta(z)dz = \begin{cases} 1, & x \geq 0; \\ 0, & x < 0; \end{cases}$$

S_{land} and S_{ocean} are the areas of land and oceans respectively; $S_{land} / (S_{land} + S_{ocean}) \cong 0.29$.

It is noteworthy that the pronounced minimum at $r_{eff} \approx 14\ \mu k$ appears in the solution of Eq. (5) (Fig. 2). This minimum can be seemingly associated with the so-called precipitation threshold [21]. In further considerations, we take into account the left-hand portion (with respect to the minimum) only of this relation.

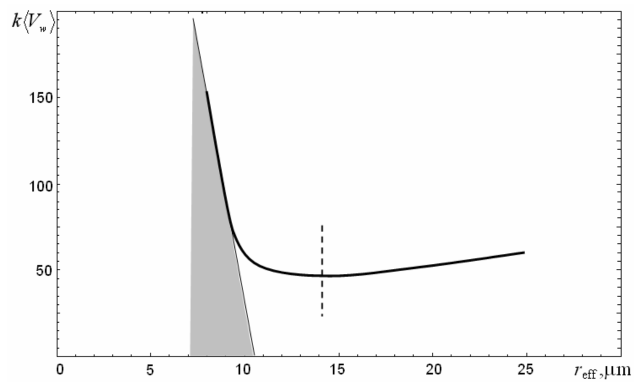

Fig. 2. Mean total volume of liquid water $\langle V_w \rangle$ in the atmosphere as a function of cloud droplet effective radius (CDR). Minimum at $r_{eff} \approx 14$ μm (vertical dotted line) corresponds to the so-called precipitation threshold. Left-hand part with respect to the minimum (hatched) is defined by the inverse linear dependence $\langle V_w \rangle$ from CDR.

Equation (5) can be reduced since the actual variations of temperature, ΔT, which are assigned to the "warm" and "ice" ages of the Earth climate, lie in the relatively narrow range. Note that the temperature fluctuations derived from EPICA Dome C [5] and Vostok ice core [6] are ±(4÷6) K. Taking under consideration this fact, it is not difficult to show on the basis of experimental data [7,8], that to such increments ΔT correspond (due to the inverse linear dependence [7,8]) small increments of mean CDR: $\Delta r_{eff} \sim$~2 to 3 μm.

Therefore, it can be supposed that the actual scenarios of global climate "know" and "feel" the relatively small range of the CDR values from a quantity of permitted values ($r_{eff} \approx 8$ to 14 μm) lying in the left of the precipitation threshold line in Fig.2. This property of climatic scenarios allows, in turn, reducing the expression for the averaged total volume of liquid water in the atmosphere. Then, a first approximation of (5) can be written as an inverse linear dependence on the CDR (see Fig. 2) or as a direct one from the temperature

$$\langle V_w \rangle \approx k_w \left(a - b r_{eff}\right) = a_w + b_w T \,. \quad (6)$$

Now, the following procedure to calculate the total water (vapour and liquid) in the atmosphere, $\langle V_{w+v} \rangle$ can be offered. There is a general argument to suppose that the theoretical dependence of water vapour volume $\langle V_v \rangle$ on the temperature is the same as (6), but this dependence differs in quantitative characteristics only. This allows supposing that the total volume of water (vapour and liquid) in the atmosphere $\langle V_{w+v}\rangle$ is directly proportional to surface temperature

$$\langle V_{w+v}\rangle = \langle V_w\rangle + \langle V_v\rangle \sim T\,. \qquad (7)$$

Therefore, if (6) is linear with regard to the temperature, in view of (7) the dependence for the total volume of water vapour $\langle V_v\rangle$ must also be linear with regard to the temperature. Then (7) can be rewritten as follows

$$\langle V_{w+v}\rangle \approx a_{wv} + b_{wv}T\,. \qquad (8)$$

As it was mentioned earlier, the cosmic ray effect and the indirect effect of aerosols on cloud are the similar in that both are driven by change in aerosol number [9]. Although these effects, in our opinion, are connected by a common "microphysics", (1) not contains a term responsible for the cosmic ray effect. In further paragraphs, we try to examine reasons why such a term is absent in (1) and to "rehabilitate" it in the general case of various time intervals.

Such a view of (8) is the direct consequence of basic assumption of our model: we assume that the temperature of ECS is defined by not single controlling parameter, but by two ones - the insolation variations and GCR intensity fluctuations (or fluctuations of Earth's magnetic field). In the next section, we consider the energy-balance model of Earth's climate with two controlling parameters.

3. Catastrophe Theory and Energy-Balance Model of Global Climate

By act of energy conservation law, the actual heat rate of Earth's radiation is approximately equal to the difference between the rate of long-wave radiation of Earth's surface, $I(T,t)$, and the heat energy re-emitted by the liquid water, $G_w(T, t)$, water vapour, $G_v(T, t)$, and carbon dioxide, $G_{CO2}(T, t)$. With the purpose of simplification, we not consider other greenhouse gases. Since the radiant equilibrium can be achieved at time scales of 10^4 to 10^5 years, the inclusion of greenhouse effect results in the following energy-balance equations for the ECS

$$U(T,t) = P_{Sun}(t)\cdot[1-\alpha(T)] - I_{Earth}(T) + \frac{1}{2}G_w(T,t) + \frac{1}{2}G_v(T,t) + \frac{1}{2}G_{CO_2}(T,t), \qquad (9)$$

where the first member $U(T, t)$, if it is nonzero, describes a magnitude of so-called "inertial" rate of heat variations in the ECS; $P_{Sun}(t)=(1/4(1-e^2))S_0\cdot\gamma\cong(1/4)S_0\cdot\gamma$ is the heat flow of solar radiation at the top of atmosphere, W; S_0=1366.2 W/m^2 is “solar constant; $\tilde{e}$ is the eccentricity of the Earth’s elliptic orbit; α is the albedo of ECS; $I_{Earth}=\gamma\delta(\sigma T^4)$, W; δ=0.95; σ is the Stephen-Boltzmann constant, W/m$^2K^4$; T is the temperature of Earth's surface, K; γ is the area of atmosphere outer boundary, m^2; t is the time, for which the energy balance is considered.

First, examine the question on functional dependence for the rate of heat energy $G_w(T, t)$ re-emitted by the liquid water on the temperature. It is obvious that (6) for mean total volume of liquid water in the atmosphere allows writing down the following relation for the rate of re-emission

$$G_w(T,t) = \varepsilon_w\rho_w\theta\langle V_w\rangle, \qquad (10)$$

where ε_w is the mean radiant power per the unit mass of liquid water, ρ_w is the density of liquid water, $\theta=<\Delta V_w>/<V_w>$ is the share of the near-surface liquid water volume in the clouds, which effectively reradiates in the atmosphere the earlier absorbed (in the long-wave range) solar energy. Obviously, due to the finite but small in magnitude length of self-absorption of the reradiated energy in the clouds, the share of the effectively reradiated liquid water volume will be inversely proportional to the complete liquid water volume $\langle V_w\rangle$ in the clouds, and hence, directly proportional to the change in the cloud covered area $\Delta\Pi_{cloud}$. But then, due to the validity of the experimental law of Svensmark and Friis-Christensen [1], the following approximate equation can be written

$$\theta\langle V_w\rangle = \langle\Delta V_w\rangle \sim \frac{1}{\langle V_w\rangle} \sim \frac{\Delta\Pi_{t=0}}{\Delta\Pi_t} \sim \frac{\Delta\Phi_{t=0}}{\Delta\Phi_t} = \Phi_\oplus^{-1}, \qquad (11)$$

where $\Delta\Pi_t/\Delta\Pi_{t=0}=\Pi_\oplus$ and $\Delta\Phi_t/\Delta\Phi_{t=0}=\Phi_\oplus$ are changes in the increment of the cloud “covered” area $\Delta\Pi_t$ and in the galactic ray intensity $\Delta\Phi_t$ in a moment of time t with respect to analogous magnitudes measured, for example, at the present time t=0.

Further on, we suppose that the mean power of reradiation of a unit mass of liquid water, ε_w, to a first approximation depends linearly on ECS temperature. Then, taking into account (11) we introduce in (10) the linear dependence ε_w on ECS temperature:

$$G_w(T,t) = \frac{\gamma h}{\langle V_{atm}\rangle}\varepsilon_w\rho_w\theta\langle V_w\rangle = \frac{\gamma h}{\langle V_{atm}\rangle}\rho_w\left(a_{w\varepsilon}T^2 + b_{w\varepsilon}T + c_{w\varepsilon}\right)\Phi_\oplus^{-1}(t), \qquad (12)$$

where h is mean height of the atmosphere, $\langle V_{atm}\rangle\approx\gamma h$.

It is necessary to consider here the details and difficulties in calculating the dependence of cosmic ray intensity on time. First, when calculating the intensity, it has to be taken into account that in the highest layers there are two factors exerting effect on it:

a) modulations determined by solar wind (this effect is in correlation with solar activity and exhibits strong temporal dependence at time scales ≥10 years, as well as in the important case for us at the millennial time scale it is correlated with the Earth's eccentricity);

b) cutting the low-energy part of cosmic ray spectrum at the expense of the geomagnetic fields (this effect depends on the broadness of the locality and its dependence on time is sufficiently small).

Second, solar wind slows down the cosmic rays. This effect is usually described by the diffusion convection model, which leads to the following formula for the observed spectrum $I(p,r,t)$ [10]

$$\Phi_{\oplus}(p,r,t)=\frac{I(p,r,t)}{I(p)}=\exp\left[-\int_{r_{\min}}^{r_{\max}}\frac{\upsilon(t)}{D(p,r',t)}dr'\right], \quad (13)$$

where r_{min} is the distance from the Earth, r_{max} is the distance of solar wind from the Sun, $\upsilon(t)$ is solar wind velocity , D is diffusion coefficient , $I(p)$ is spectrum in interstellar space depending on the p particle momenta.

This effect becomes maximal both in years of maximum solar activity at time scales ≥10 years and in the time moments of minimal eccentricity of the Earth's orbit at the millennial time scale. For example, a proton ray with energy of 1 GeV turns to be twice bigger for minimal solar activity than in the case of maximal solar activity. This effect is decreased to <10% for E_p=10 GeV, which, by the way, is shown in the experiments of Svensmark and Friis-Christensen [1].

Unfortunately, it is obvious that the calculation on the basis of (13) or the use, for example, of the geophysical reconstruction data (according to the traces in meteorites) of the temporal evolution of cosmic ray intensity at the millennial time scale is practically impossible at present. For this reason, it seems that the necessary verification of the global climate model with two governing parameters (insolation and cosmic ray intensity) by comparing the model solutions and the known experimental time series of palaeotemperature (e.g., the Vostok ice core data [6] over the past 420 kyr and the EPICA ice core data [5] over the past 730 kyr), becomes, at first glance, insurmountable problem. In our opinion, however, there is one "hinge" allowing the encompassment of this problem, although approximately, with certain limitations. This is done in the following manner.

It is known that the intensity of galactic cosmic rays, reaching the middle troposphere, is modulated by solar wind. However, when solar wind "sweeps" to one extent or another, the galactic protons, it excites at the same time to one extent or another the Earth magnetic field due to the magnetic reconnection phenomenon [5]. In other words, the higher the solar wind magnetic strength, the higher the relative reduction of intensity of cosmic rays reaching the middle troposphere and effectively participating in cloud formation, and the higher the relative increase of the Earth magnetic field. If these physically determined relations are expressed in terms of formulas, the following approximate inverse relation will be obtained for the dependence of the relative changes in intensity $\Phi_{\oplus}$ of galactic cosmic rays and the relative changes $H_{\oplus}$ of the Earth's magnetic field:

$$\Phi_{\oplus}(t)=\frac{\Delta\Phi_t}{\Delta\Phi_{t=0}}\sim\frac{H_{t=0}}{H_t}=H_{\oplus}^{-1}(t), \quad H_{\oplus}(t)\geq 0.5, \quad (14)$$

where $H_t/H_{t=0}=H_{\oplus}$ is changes in the Earth's magnetic field H_t in time t, with respect to analogous magnitude, measured in the present time t=0.

It has to be reminded again that (14) represents rough approximation. Our aspiration of achieving the transformation by means of (14) from the temporal sample of $\Phi_{\oplus}$ values to the analogical sample of $H_{\oplus}$ values, is explained by the remarkable circumstance that the temporal sample of $H_{\oplus}$ values may be determined on the basis of experimental magnetic palaeodata of Yamasaki and Oda **[11]**. For example, the temporal evolution of the relative changes $H_{\oplus}(t)$ of the Earth's magnetic field at the millennial time scale can be calculated using the expression

$$H_{\oplus}(t)=\frac{H_t}{H_{t=0}}=\frac{M_t\chi_0}{\chi_t M_0}, \quad (15)$$

where $M=\chi H$ is magnetic moment per unit volume or magnetization; χ is magnetic susceptibility.

The necessary experimental data connected with the measurements of the magnetization M_t and magnetic susceptibility χ_t at the millennial time scale, have been obtained and presented in the work of Yamasaki and Oda [11] for a time period $t\in[0, 2.25]$ million years.

In this way, taking into account (14), the expression (12) for the energy of liquid water in the clouds assumes the following form:

$$G_w(T,t)=\frac{\gamma h}{\langle V_{atm}\rangle}\rho_w\left(a_{w\varepsilon}T^2+b_{w\varepsilon}T+c_{w\varepsilon}\right)H_{\oplus}(t)\cdot \quad (16)$$

In analogical way the expression can be obtained for the power of thermal energy $G_v(T,t)$, reradiated vapour water, found in the clouds:

$$G_v(T,t)=\frac{\gamma h}{\langle V_{atm}\rangle}\rho_v\left(a_{v\varepsilon}T^2+b_{v\varepsilon}T+c_{v\varepsilon}\right)H_{\oplus}(t), \quad (17)$$

where ε_v is mean reradiation power per unit mass vapour water, W/kg; ρ_v is the density of vapour water; $\langle V_{atm}\rangle\approx\gamma h$.

To examine a question on the functional dependence for the rate of heat energy $G_{CO_2}(T,t)$ on the temperature of ECS, use the analysis of known experimental data on the variations of temperature and carbon dioxide content over the past 420 kyr from the Vostok ice core [6] and and over 730 kyr from the EPICA ice core [5]. It is obvious that these data are highly linear correlated. Therefore, it can be supposed that the dependence for the rate of heat energy on the temperature of ECS is also linear

$$G_{CO_2}(T,t) = \frac{\gamma h}{\langle V_{atm} \rangle} \varepsilon_{CO_2} \beta T , \qquad (18)$$

where ε_{CO_2} is the radiant energy of unit mass of carbon dioxide, β is the accumulation rate of carbon dioxide in the atmosphere, which is normalized at the unit of temperature, kg/K.

Theoretically this dependence can be also explained within the three-mode radiative model of the kinetics processes in the atmosphere [12]. Indeed it is provided by the energy and heat exchange processes in the mixture CO_2-N_2-O_2-H_20 of atmospheric gases interacting with electromagnetic radiation [12]. In the absorption of electromagnetic radiation by the atmospheric molecular gases a redistribution of molecules on the energy levels of internal degrees of freedom occurs and the saturation of absorption results in the changes of the absorption coefficient of gas. In our case, in fact the formation and accumulation of the excited molecules of nitrogen owing to the resonant transfer of excitation from the molecules CO_2 results in the change of environment polarizability, but conserves the linear dependence of the rate of heat energy on the temperature of ECS.

It must be added that the dependence for the effective value of albedo on the temperature of ECS is chosen as the continuous parameterization

$$\alpha = \alpha_0 - \eta_\alpha \cdot (T - 273) . \qquad (19)$$

Equation (19) represents well, for example, the behavior of albedo (under α_0=0.5360, η_α=0.01513 K^{-1}) in the temperature range of 282 to 290 K.

Finally, assembling all partial contributions of heat fluxes (16)-(19) and $I_{Earth} = \gamma\delta(\sigma T^4)$ into the finite energy-balance expression (9), we derive

$$U^*(T,t) = \frac{1}{4}T^4 + \frac{1}{2}a(t) \cdot T^2 + b(t) \cdot T , \qquad (20)$$

where

$$a(t) = -\frac{1}{4\delta\sigma} a_\mu H_\oplus(t) , \qquad (21)$$

$$b(t) = -\frac{1}{4\delta\sigma}\left[\frac{\eta_\alpha S_0}{4} + \frac{1}{2}\beta + \frac{1}{2} b_\mu H_\oplus(t)\right] , \qquad (22)$$

$$U^*(T,t) = \frac{1}{4\delta\sigma}[\frac{1}{4}(1-\alpha_0 - 273\eta_\alpha)S_0 + \frac{1}{2} b_{CO_2} h/\langle V_{atm} \rangle \qquad (23)$$

$$+\frac{1}{2}c_\mu H_\oplus - U(T,t)],$$

$$a_\mu = k_H(\rho_w a_{w\varepsilon} + \rho_v a_{w\varepsilon})\gamma^{-1} , \quad [W/m^2K^2],$$

$$b_\mu = (\rho_w b_{w\varepsilon} + \rho_v b_{v\varepsilon})\gamma^{-1} , \quad [W/m^2K],$$

$$c_\mu = (\rho_w c_{w\varepsilon} + \rho_v c_{v\varepsilon})\gamma^{-1} , \quad [W/m^2].$$

where $a_{w\varepsilon}$, $a_{v\varepsilon}$, $b_{w\varepsilon}$, $b_{v\varepsilon}$, $c_{w\varepsilon}$, $c_{v\varepsilon}$ are constant coefficients with dimensions determined by (17), (18) and (19), respectively

It is obvious that (20) describes the collection of energy-balance functions $U^*(T,a,b)$, which depend on two controlling parameters, $a(t)$ and $b(t)$. Also, this collection represents so-called potential of fold catastrophe [13].

In future, we will be interested by the type of the "excited" equation (20) or, more exactly, the equation of fold catastrophe (20) relative to the increment $\Delta T = T - T_0$ of the following view: $U(T_0+\Delta T,a,b) - U(T_0,a,b) = \Delta U$, where T_0 is the mean ECS temperature averaged at the respective time interval Δt. Also, the increment for the first right-hand term in (20) can be present in the following equivalent form:

$$(T_0 + \Delta T)^4 - T_0^4 \cong 7 \cdot 10^{-3} \cdot T_0^3 \cdot (\Delta T)^4 + 4 \cdot T_0^3 \cdot \Delta T \text{ at } \Delta T = 0 \div 4K, \qquad (24)$$

for which the mean error of approximation at given range of temperature not exceeds 0.01%.

Let us remind that the normalized variations of insolation,

$$\Delta W = \frac{W - \langle W_0 \rangle}{\sigma_S} , \qquad (25)$$

with mean value $<\Delta W>=0$ and dispersion $\sigma^2_{\Delta W}$ =1 is applied more often for the simulation of the ECS.

Deriving an equation in the form (20) with respect to ΔT, the following expression for the increment of heat rate ΔU^* can be defined

$$\Delta U^*(\Delta T,t) \cong \frac{1}{4}\Delta T_t^4 + \frac{1}{2}\tilde{a}(t) \cdot \Delta T_t^2 + \tilde{b}(t) \cdot \Delta T_t \qquad (26)$$

where

$$\tilde{a}(t) = -\frac{37.6}{\sigma T_t^3} a_\mu H_\oplus(t) = -\tilde{a}_0 \cdot H_\oplus(t) \qquad (27)$$

$$\tilde{b}(t) = -\frac{37.6}{\sigma T_t^3}\left[\eta_\alpha \frac{S_0 + \Delta\hat{W}(t)\sigma_S}{4} - 4\delta\sigma T_t^3 + \frac{1}{2}\beta + \frac{1}{2}(2a_\mu T_t + b_\mu)H_\oplus(t)\right] =$$

$$= -\tilde{b}_0\left[\eta_\alpha W_{reduced}(t) - 4\delta\sigma T_t^3 + \frac{1}{2}\beta + \frac{1}{2}(2a_\mu T_t + b_\mu)H_\oplus(t)\right], \qquad (28)$$

where S_0 is "solar constant", $\Delta\hat{W}(t)$ is reduced normalized variations of insolation, σ_S is reduced mean square deviation,

$4W_{reduced}$= S_0+ $\Delta\hat{W}(t)\,\sigma_S$ is reduced mean annual insolation of the Earth.

And, finally, the canonical form of the variety of the fold catastrophe, which represents a set of points (T,a,b) and $(\Delta T,\tilde{a},\tilde{b})$ satisfies the equation

$$\frac{\partial}{\partial T}U^*(T,t)=T_t^3+a(t)\cdot T_t+b(t)=0 \qquad (29)$$

$$\frac{\partial}{\partial(\Delta T)}\Delta U^*(\Delta T,t)\cong\Delta T_t^3+\tilde{a}(t)\cdot\Delta T_t+\tilde{b}(t)=0\,. \qquad (30)$$

Thus the general bifurcation problem contained in the arriving at a solution $T(t)$ and the excited solution $\Delta T(t)$ is reduced to the determination of the solutions set of Eq. (29)-(30) for the appropriate joint trajectory $\{a(t),b(t)\}$ and $\{\tilde{a}(t),\tilde{b}(t)\}$ in the space corresponding to the controlling parameters.

4. NUMERICAL EXPERIMENT

It is obvious that in order to solve the bifurcation equations (27) and (28) which describes extreme increments of temperature T and ΔT, respectively, it is necessary to determine the values of three climatic constants a_μ , b_μ and β (in (27) and (28)). The values of other constants are known.

The value of β is determined by means of known estimation for the rate of heat energy re-emitted by the carbon dioxide, W_{CO_2} ~1.7 W/m^2 [14], in which the sum of the variations of re-radiation energy $\Delta G_{CO_2}(T_{t=0})$ in (29) and $\Delta G_{CO_2}(\Delta T_{t=0})$ in (30) in linear approximation correspond to the following equation:

$$\frac{1}{\gamma}\Delta G_{CO_2}(T_{t=0})+\frac{1}{\gamma}\Delta G_{CO_2}(\Delta T_{t=0})=\frac{1}{2}\beta T_{t=0}= \\ =\frac{1}{2}W_{CO_2}\approx\frac{1}{2}\cdot 1.7\quad [W/m^2]. \qquad (31)$$

where $H_\oplus(t=0)=1$; the coefficient equal to 1/2 allows for an isotropy of radiation by the carbon dioxide. Further on the initial condition will be taken into account, for which the value of the modern climatic representative temperature T_0 is about

$$T_0=T_{t=0}+\Delta T_{t=0}=288.6\pm1.0,\quad K \qquad (32)$$

Hence from (40) with consideration for (29) and (32), it is obtained

$$\beta\approx\frac{1.7}{T_{t=0}}\cong 0.006,\quad [W/m^2K] \qquad (33)$$

As a result of simulation of the solution of the system of the initial equation (29) and the "excited" equation (30) with taking into account the initial conditions (32)–(33), the following values have been determined for the climatic constants a_μ and b_μ:

$$a_\mu=0.222,\quad b_\mu=-127.249\,. \qquad (34)$$

and the initial parameters of the equation system (29)-(30):

$$T_{t=0}=286.031\ \ K,\quad \Delta T_{t=0}=2.100\ \ K,\quad \beta=0.006\ \ W/m^2K\,. \qquad (35)$$

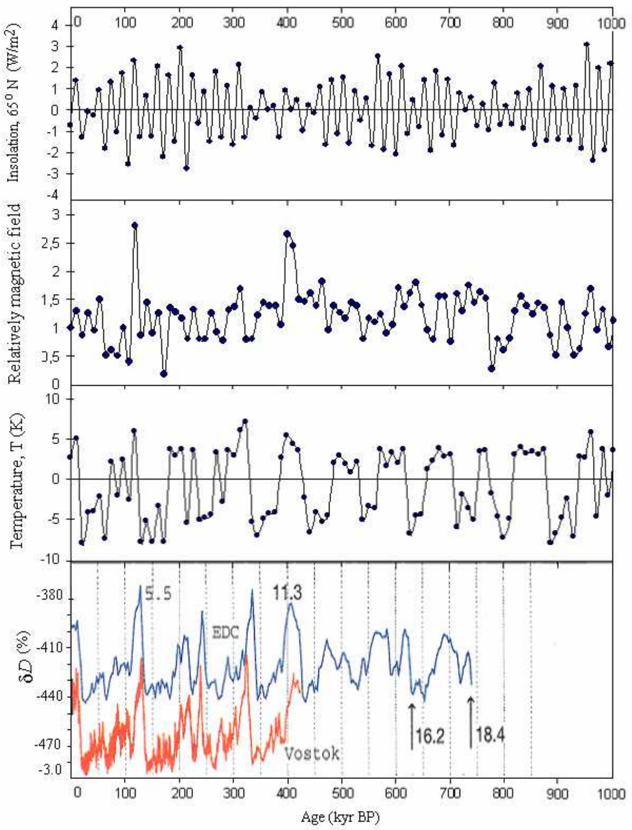

Fig. 3. Model of climatic response insolation and magnetic field variations of the past 730 kyr compared with isotopic temperature data on climate of the past 730 kyr. Variations insolation (*a*) at 65°N at the summer solstice over the past 730 kyr. Variations of magnetic field of Earth (*b*) over the past 730 kyr [11]. EPICA [6] and Vostok [5] time series of isotopic temperature ΔT_S (*d*) at the surface and result of our model (*c*) calculated by (28): evolution of the increment of temperature ΔT relative to the average temperature T_0=286 K over the past 1000 kyr.

The following remark has to be mentioned here. The investigation of the parameterized solution (of the temperatures T_t+ΔT_t) of the equation system (29)-(30) exhibits high degree of solution stability with respect to relatively small "excitations" of the initial and boundary parameters (34)-(35). In other words, the small variations of the climatic constants a_μ and b_μ lead to small changes in the initial parameters $T_{t=0}$, $\Delta T_{t=0}$ and β. A little in advance, it is important to note that the general solution of the equation system (29)-(30) or, more exactly, the distribution of the theoretical paleotemperatures $T_t+\Delta T_t$ in time, as shown by the numerical experiment, practically does not change its form at small variations of the climatic constants (34) and initial parameters (35). It means that the approximate invariance of the form of the theoretical temperature ($T_t+\Delta T_t$) distribution with time is predetermined by a certain attractor in the phase portrait of the physical system (29)-(30) and needs a special physical-mathematical substantiation, but this is already another problem, going beyond the scope of the present work.

Thus the high goodness of fit between the experimental (Fig. 3d) and theoretical (Fig. 3c) data is a validity indicator of main assumption used in our model; we assume that the temperature of ECS is caused both the variations of insolation and variations of GCR intensity (or, equivalently, variations of Earth magnetic field).

It can be concluded from the above mentioned that the most important, in our opinion, statement of presented model is the fact that the Earth climate, on the one hand, is completely defined by the two controlling parameters - insolation and galactic cosmic rays - and, in the other hand, is quite predictable on the millennial time scales if only theoretical or experimental values on long-term variations of relative palaeointensity $H_\oplus(t)$ are present.

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