Calculation of differential temperature coefficients for muons at different zenith angles

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***Abstract*—Influence of atmospheric temperature on muon flux at sea level is considered. Results of calculations of the differential temperature coefficients (DTC) for muons at different zenith angles and threshold energies are presented. These calculations are based on the solution of diffusion equations for mesons and muons in atmosphere. In calculations, a six-layer stationary spherical model of atmosphere was used, contributions of both pions and kaons were taken into account. Also for muons, relation between energy loss and muon energy and density of air was taken into account. Comparison of results of DTC calculations with results of earlier works shows only qualitative agreement; possible sources of the differences are discussed.**

1. Definition and Calculation of Differential Temperature Coefficients

IN investigations of muon flux variations at the Earth surface, barometric and temperature effects have to be taken into account. For calculation of temperature effect correction, it is necessary to know differential temperature coefficients, which make it possible to correct counting rate taking into account changes of the temperature at all altitudes of the atmosphere. Let us denote $N(E_{\min},X,\theta)$ as integral muon intensity at observation level X (in atm) for zenith angle θ and threshold energy $E_{\min}$. If atmospheric temperature is changed by $\Delta T(h)$ (h is the atmospheric depth in atm) muon flux will be changed by $\Delta N(E_{\min},X,\theta)$ and one can write the relative change of the muon flux in a following way [1]:

$$\Delta N(E_{\min},X,\theta)/N(E_{\min},X,\theta)\cdot 100\,\% = \int_0^X W_{\mathrm{T}}(E_{\min},X,h,\theta)\Delta T(h)dh, \quad (1)$$

here the function $W_{\mathrm{T}}(E_{\min},X,h,\theta)$ is DTC.

If the altitude dependence of atmospheric temperature $T(h)$ and "standard" value of muon integral intensity are known, with the help of DTC it is possible to calculate the intensity corrected for the temperature effect N^{corr}:

$$N^{\mathrm{corr}}(E_{\min},X,\theta) = N^{\exp}(E_{\min},X,\theta) - \Delta N(E_{\min},X,\theta), \quad (2)$$

where $N^{\exp}$ is the experimental muon integral intensity.

DTC can be found on the basis of equations for muon spectrum calculations. In turn, calculations of muon integral intensity at the atmospheric depth X are based on the solution of diffusion equations for mesons and muons in the atmosphere [2]:

$$N(E_{\min},X,\theta) = \int_{E_{\min}}^{\infty} dE \int_0^X dz \int_0^z dz' \int_{\varepsilon}^{(\eta c^2/\mu c^2)^2\varepsilon} dE_\eta\, F(E,X,\theta,z,z',E_\eta), \quad (3)$$

$$F(E,X,\theta,z,z',E_\eta) = A_\eta \cdot \exp\left(\frac{-z'}{L_p}\right)\cdot E_\eta^{-\gamma}\cdot \frac{B_\eta}{(1-(\mu c^2/\eta c^2)^2)E_\eta}\cdot\exp\left(-\frac{z-z'}{\lambda_\eta} - \frac{l(z)-l(z')}{c\tau_\eta}\cdot\frac{\eta c^2}{E_\eta}\right)\cdot \frac{\eta c^2}{c\tau_\eta \rho(z) E_\eta}\cdot\frac{d\varepsilon(E,X-z)}{dE}\cdot\exp\left(-\int_z^X \frac{\mu c^2 dt}{c\tau_\mu \varepsilon(E,X-t)\rho(t)}\right). \quad (4)$$

Here E is muon energy at observation level X, E_η is the energy of produced meson (π or K), z and z' are depths of muons and mesons generation along the track (in g/cm^2), ε is muon energy at depth z, η and μ are masses of meson and muon correspondingly, c is the velocity of light, τ_η and τ_μ are life-times correspondingly of meson and muon, A_η is the normalization constant (A_π/A_K = 0.15), L_p is the absorption length of primary nucleons in air, γ is the index of generation function of mesons, λ_η is interaction mean free path of meson, ρ is air density, B_η is the probability of corresponding mode of two-body decay (B_π = 1.0, B_K = 0.64), $l(z) = \int_{\mathrm{const}>0}^{z} dt/\rho(t)$.

In order to obtain the expression for $\Delta N(E_{\min},X,\theta)$, it is necessary to vary the function $N(E_{\min},X,\theta)$ with respect to temperature at constant atmospheric pressure P:

$$\Delta N(E_{\min},X,\theta) = \int_{E_{\min}}^{\infty} dE \int_0^X dz \int_0^z dz' \int_{\varepsilon}^{(\eta c^2/\mu c^2)\varepsilon} dE_\eta\, F(E,X,\theta,z,z',E_\eta) \cdot\left(\frac{\delta T(z)}{T(z)} - \frac{\eta c^2 R}{c\tau_\eta E_\eta M}\int_{z'}^{z}\frac{\delta T(t)dt}{P(t)} - \frac{\mu c^2 R}{c\tau_\mu M}\int_z^X \frac{\delta T(t)dt}{\varepsilon(E,X-t)P(t)}\right), \quad (5)$$

here M is molecular weight of air, R is universal gas constant. This formula is in agreement (up to dimensions of quantities) with formula presented by L.I.Dorman and V.G.Yanke [1]. First two terms in parentheses in the integral reflect the change of muon flux because of change of pions and kaons decay probability; the third term reflects the variation of muon flux because of changes of muon decay probability. Thus, the temperature effect can be divided in two components (so-called meson effect and muon effect):

$$W_{\mathrm{T}}(E_{\min},X,h,\theta) = W_{\mathrm{T}}^{\eta}(E_{\min},X,h,\theta) + W_{\mathrm{T}}^{\mu}(E_{\min},X,h,\theta). \quad (6)$$

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To obtain $W_T(E_{min},X,h,\theta)$ it is necessary to combine equations (1) and (5), then to take $\Delta T(h)$ equal to $\delta(h)$. After transformation we obtain formulas for DTC calculation:

$$W_T^{\eta}(E_{min},X,h,\theta)=\frac{100\%}{N(E_{min},X,\theta)}\cdot$$
$$\cdot\left[\frac{1}{T(t_0)}\int_{E_{min}}^{\infty}dE\int_0^{t_0}dz'\int_{\varepsilon}^{(\eta^2/\mu^2)\varepsilon}dE_{\eta}\,F(E,X,\theta,t_0,z',E_{\eta})+\right.$$
$$+\int_{E_{min}}^{\infty}dE\int_{t_0}^{X}dz\int_0^{t_0}dz'\int_{\varepsilon}^{(\eta^2/\mu^2)\varepsilon}dE_{\eta}\,F(E,X,\theta,z,z',E_{\eta})\cdot \qquad (7)$$
$$\left.\cdot\left(-\frac{1}{c\tau_{\eta}}\frac{\eta c^2}{E_{\eta}}\frac{R}{M\,P(t_0)}\right)\right],$$

$$W_T^{\mu}(E_{min},X,h,\theta)=\frac{100\%}{N(E_{min},X,\theta)}\cdot$$
$$\cdot\int_{E_{min}}^{\infty}dE\int_0^{t_0}dz\int_0^{z}dz'\int_{\varepsilon}^{(\eta^2/\mu^2)\varepsilon}dE_{\eta}\,F(E,X,\theta,z,z',E_{\eta})\cdot \qquad (8)$$
$$\cdot\left(\frac{-\mu c^2R}{c\tau_{\mu}M\varepsilon(E,X-t_0)P(t_0)}\right).$$

The path t_0 along the trajectory of the particle (in g/cm^2) corresponds to the depth h.

The sign of the meson effect W_T^{η} is positive, since if the temperature of atmosphere increases, the atmosphere expands, density of air decreases and the probability of the interaction of mesons (kaons and pions) at unit of geometric path becomes smaller, hence decay probability becomes higher. Sign of the muon effect W_T^{μ} is negative, because if the temperature of atmosphere increases and atmosphere expands, the geometric path from generation level to registration one becomes longer, so higher number of muons will decay. The relation of absolute values of the effects depends on E_{min}. In case of low threshold energies the absolute value of the muon effect W_T^{μ} is greater than the value of meson effect W_T^{η}, and the sign of the total effect W_T is negative. In case of high threshold energies, the muon effect degrades (muons have no time to decay in the atmosphere) and the sign of the total effect becomes positive.

In our calculations the next values of parameters were used: $\gamma = 2.7$, $L_p = 110$ г/см^2, $\lambda_{\pi} = 120$ г/см^2 и $\lambda_K = 150$ г/см^2. Results of W_T^{μ} and W_T^{η} calculations for $E_{min} = 0.4$ GeV and six values of zenith angle are presented in Fig.1. As it is seen, W_T^{μ} is negative, W_T^{η} is positive and the absolute value of the muon effect W_T^{μ} is greater than value of meson effect W_T^{η}. At $h > 0.7$ atm contribution of meson effect to the total effect is negligible because of small number of mesons at these depths. Total effect W_T is presented in Fig.2 and Table 1. Example of dependence of DTC on atmospheric depth h for $\theta = 0°$ and six values of threshold energy is shown in Fig.3.

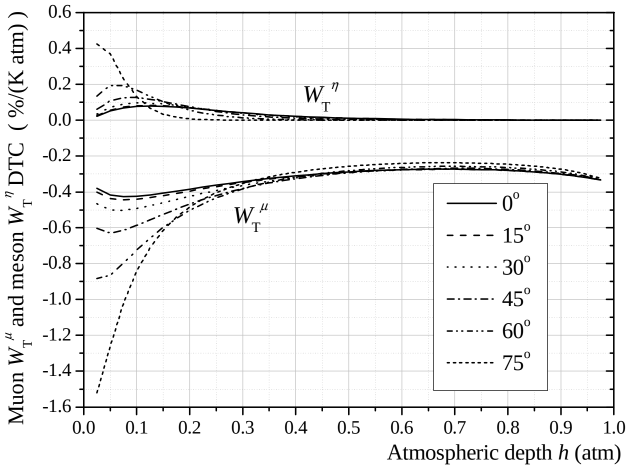

Fig. 1. Muon W_T^{μ} and meson W_T^{η} differential temperature coefficients calculated for $E_{min} = 0.4$ GeV and several zenith angles.

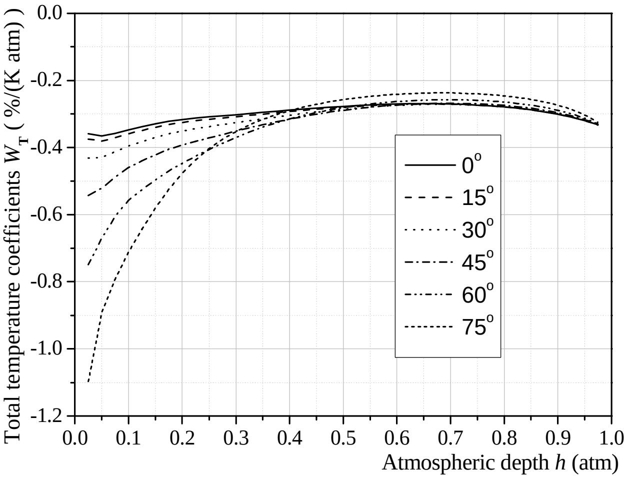

Fig. 2. Total differential temperature coefficients W_T calculated for $E_{min} = 0.4$ GeV and several zenith angles.

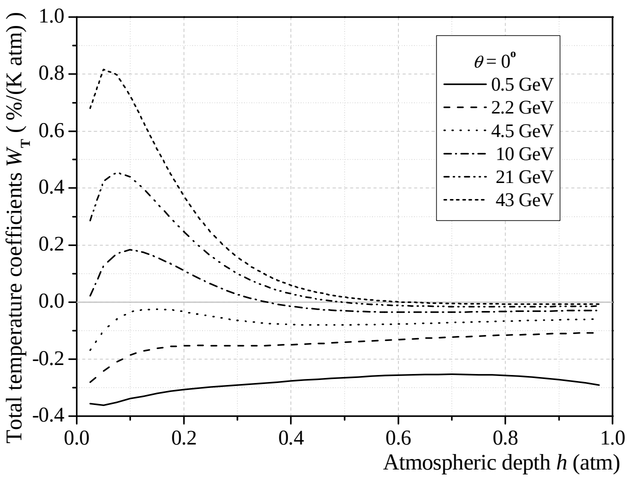

Fig. 3. Total differential temperature coefficients W_T calculated for $\theta = 0°$ and several values of threshold energy.

TABLE I

TOTAL DIFFERENTIAL TEMPERATURE COEFFICIENTS
FOR E_{MIN} = 0.4 GEV AND SIX VALUES OF ZENITH ANGLE

h, atm	W_T, % / (K atm)					
	θ = 0°	θ = 15°	θ = 30°	θ = 45°	θ = 60°	θ = 75°
0.05	-0.366	-0.381	-0.429	-0.521	-0.670	-0.892
0.10	-0.347	-0.359	-0.395	-0.460	-0.557	-0.711
0.15	-0.330	-0.340	-0.370	-0.422	-0.497	-0.580
0.20	-0.317	-0.325	-0.351	-0.393	-0.447	-0.476
0.25	-0.309	-0.316	-0.338	-0.371	-0.406	-0.403
0.30	-0.302	-0.309	-0.326	-0.351	-0.370	-0.352
0.35	-0.295	-0.300	-0.315	-0.332	-0.339	-0.316
0.40	-0.288	-0.293	-0.304	-0.315	-0.315	-0.290
0.45	-0.282	-0.286	-0.294	-0.301	-0.296	-0.271
0.50	-0.277	-0.280	-0.286	-0.289	-0.281	-0.257
0.55	-0.273	-0.275	-0.279	-0.280	-0.270	-0.248
0.60	-0.270	-0.272	-0.274	-0.273	-0.263	-0.241
0.65	-0.269	-0.270	-0.272	-0.269	-0.259	-0.238
0.70	-0.270	-0.271	-0.271	-0.268	-0.258	-0.237
0.75	-0.273	-0.274	-0.274	-0.270	-0.259	-0.240
0.80	-0.279	-0.279	-0.278	-0.274	-0.264	-0.246
0.85	-0.288	-0.288	-0.287	-0.283	-0.274	-0.256
0.90	-0.301	-0.301	-0.300	-0.296	-0.289	-0.274
0.95	-0.320	-0.320	-0.319	-0.317	-0.312	-0.303

2. COMPARISON WITH RESULTS OF EARLIER WORKS

Differential temperature coefficients calculated by L.I.Dorman and V.G.Yanke [1] for E_{min} = 0.4 GeV and four values of zenith angle are shown in Fig.4. DTC obtained in [1] for large zenith angles (Fig.2) are in qualitative agreement with our results. But for zenith angle $\theta = 0°$, the dependence of $W_T(h)$ differs very much. It should be noted however that a direct comparison of DTC obtained in two works is not quite justified because in calculations [1] different values of parameters were used (γ = 2.5, L_p = 120 g/cm^2, λ_π = 60 g/cm^2), contribution of kaons to muon flux was not taken into account, muon energy loss was considered as constant (2 MeV·g^{-1}·cm^{-2}), and simplified model of atmosphere was used.

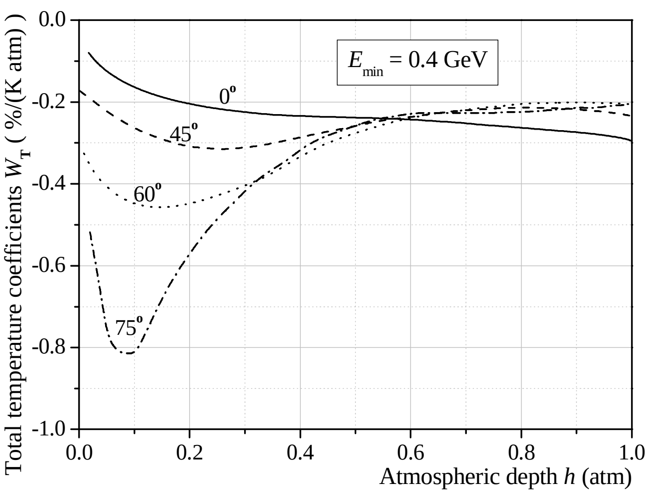

Fig. 4. Total differential temperature coefficients W_T for several values of zenith angle calculated in [1].

To understand the source of disagreement in the dependence of $W_T(h)$, additional calculations were carried out. Approximations and values of parameters used in [1] were introduced in our model of DTC calculations, contribution of kaons was switched off. Results of additional calculations are presented in Fig.5. Comparison of DTC obtained in [1] and Fig.5 shows that disagreement in dependence $W_T(h)$ for $\theta = 0°$ remains. Differences between two calculation results, probably, can be related with insufficient precision of numerical calculations in [1].

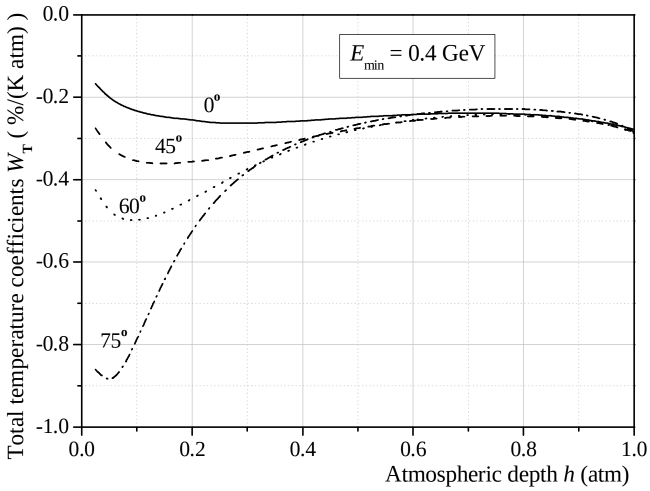

Fig. 5. Total differential temperature coefficients W_T for several values of zenith angle calculated in our work with approximations used in [1].

In work of K.Maeda [4], an empirical formula for muon production spectrum was used for DTC calculations: $P^{\eta\mu}(p,z) = A/(a' + r(p))^{3.38}\exp(-z/L_n)$, here z is the depth of muons generation along the track (in g/cm^2), r is the residual range in air (in g/cm^2) of muons with momentum p, A and a' are constants, L_n = 120 g/cm^2 is attenuation mean free path of primary particles. In calculations, two-layer stationary spherical model of atmosphere was used, dependences of meson and muon decay probability on atmospheric temperature, and of energy loss on muon energy and air density were taken into account. DTC calculated in [4] for $\theta = 0°$ and six values of threshold energy (0.5, 2.2, 4.5, 10, 21 and 43 GeV) are presented in Fig.6. In order to compare our results with results of [4], calculations of DTC by formulas (7) and (8) for these threshold energies were carried out (Fig.7). As it is seen, the comparison of calculation results of our work with calculations [4] shows a qualitative agreement, however quantitatively the results differ by tens percent. Difference, probably, can be caused by approximations used in [4].

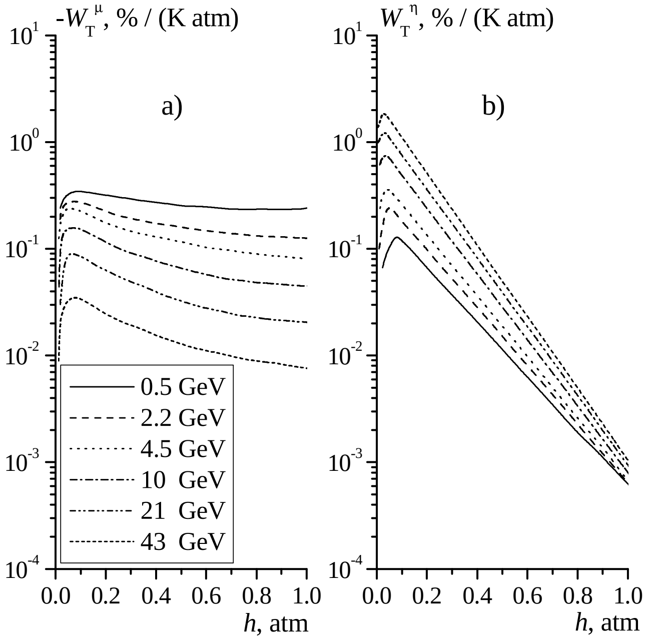

Fig. 6. Muon W_T^μ (a) and meson W_T^η (b) differential temperature coefficients calculated in [4] for $\theta = 0°$ and for several values of threshold energy.

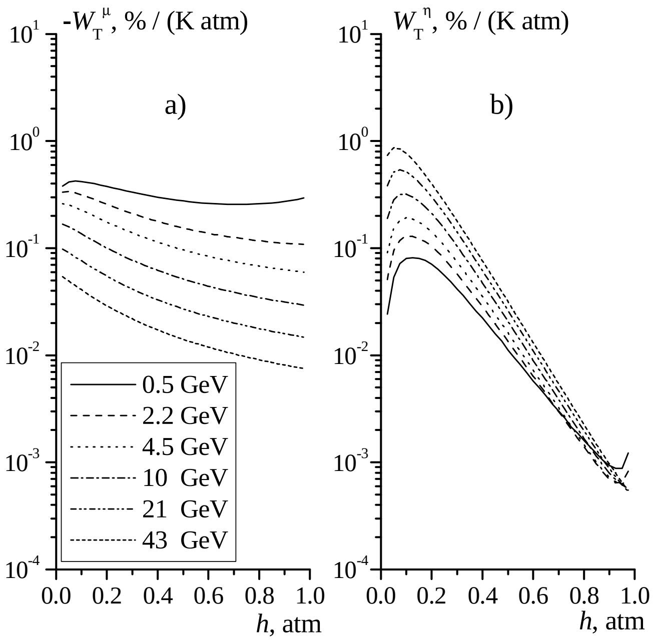

Fig. 7. Muon W_T^μ (a) and meson W_T^η (b) differential temperature coefficients for several values of threshold energy calculated in our work.

3. Conclusion

Differential temperature coefficients were calculated for six-layer spherical model of atmosphere, taking into account contributions of both pions and kaons to muon flux. Results of calculations for several values of zenith angle and threshold energy are presented.

Results of present DTC calculations are only in qualitative agreement with the preceding results. But quantitative differences amount to tens percent. These differences can be caused by approximations and insufficient precision of numerical calculations in earlier works.

Acknowledgment

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